

MASTER IN ECONOMICS: EMPIRICAL APPLICATIONS AND POLICIES

# Interest Rates forecasts: An analysis of SPF

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***Abstract:** The Federal Reserve Bank of Philadelphia publishes a quarterly survey of U.S. macroeconomic forecasts, called the Survey of Professional Forecasters (SPF). In this thesis, I have used the historical mean forecasts of the 3-month Treasury Bill yield and the 10-year Treasury Bond yield to answer three different questions about these forecasts: (i) Have professional forecasters done better in mean than a naïve forecast based on a random walk process? (ii) Have they generated efficient/rational forecasts? (iii) Have they forecasted well the trend of the interest rates? The empirical results show that for the short-term interest rate the mean of the SPF has forecasted better than a naïve forecast model. Moreover, they made good trend forecasts, but their forecast cannot be considered rational. Meanwhile, for the long-term interest rate, the professional forecasts are worse than those based on a random walk process for longer horizons than 3 quarters and the mean of SPF is not good for trend forecasting.*

## **1. Introduction**

The future is unknown and it is always so. Economists, firms, governments and central banks make estimations about the future trying to guess in advance. Looking at forecast of professional economists, who arguably have a good understanding of the overall economy, sounds as a good idea for getting ahead of oneself. The Federal Reserve Bank of Philadelphia publishes a database<sup>1</sup> with the responses of the Survey of Professional Forecasters –from now SPF-. In particular, this data bank contains the quarterly mean forecasts of many relevant variables of the US economy since 1968. The SPF asks what the quarterly mean value of major macroeconomic variables is going to be in each of the next 6 quarters.

This thesis focuses on the SPF forecasts of the 3-month Treasury Bill and the 10-year Treasury Bond yields from the third quarter of 1981 and from the first quarter of 1992, respectively. This study uses the mean of the SPF associated with the forecasts of these two interest rates up to 6-quarter horizons (i.e. 1, 2, 3, 4, 5 and 6 quarter ahead forecasts) in order to answer three important questions:

- i. Have professional forecasters made a better forecast than a naïve forecaster?
- ii. Have professional forecasters been rational/efficient?
- iii. Have professional forecasters made good trend forecasts of interest rates?

The answers to these questions allow us to know if professional forecasters have made good predictions about two important US government bond interest rates –short and long term yields-. As far as we know there is no paper in the literature assessing the rationality of short interest rates forecasts. The goal of this research is to study whether the mean forecasts of interest rates quarterly published by the Federal Reserve Bank of Philadelphia are efficient, so they provide valuable information about the future. This feature is important because if the SPF reports efficient forecasts, they are useful for taking well-informed economic decisions as well as for improving the economic modeling of expectations. Moreover, we suggest a scoring

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<sup>1</sup> The American Statistical Association and the National Bureau of Economics Research started in 1968 to collect forecasts on macroeconomic variables from professional forecasters. The Federal Reserve Bank of Philadelphia is carrying out the survey since 1990.

method for analyzing the trend expectations based on the SPF forecast mean for the two yields analyzed.

This master thesis is organized as follows: Section 2 briefly describes the related literature. Section 3 describes the data used in the analysis. Section 4 explains the three dimension of the analysis. Section 5 answers the three questions for the short term interest rate. Section 6 carries out the same analysis for the T-Bond yield. Finally, Section 7 concludes.

## **2. Previous studies**

The history of the SPF started in 1968. It was called the Quarterly Economic Outlook Surveys and it was organized by NBER-ASA. Zarnowitz (1968) described the most important reasons for the genesis of this survey. Its main objective was to carry out a survey with the predictions of professional economists whose main task was to make economic forecasts. Historical economic data is necessary to analyze how good a forecasting procedure performs. After 22 years of publishing the SPF, Zarnowitz (1993) wrote a seminal paper in this literature. His paper describes the history of the SPF, showing relevant ideas about the use of this survey for forecasting the future. A first idea was the following: “However, the value of forecasts depends not only on its accuracy and unbiasedness but also on the predictability of the variable or event concerned”. This idea simply states that for making good forecasts is also important what we are forecasting on. That is, it is easier forecasting certain macroeconomic and monetary variables than others. A second important idea is that there are two different ways of analyzing this survey: focusing on every forecaster’s forecast or in the “consensus” forecast. The second approach, which uses either the mean or the median of the survey, has the problem of not taking into account the dispersion of opinions, which is a measure of the uncertainty associated with the variable being forecasted.

The website of the Federal Reserve Bank of Philadelphia has track records of the forecast errors. These reports study how the errors have behaved depending on the variable and forecast horizon considered. These reports provided a few ideas for assessing the efficiency of professional forecasts on interest rates in this thesis, such as using a naïve forecast based on a random walk model, where the best prediction of variable is just its last observed value, and analyzing the errors in absolute value.

Rationality is one of the main features in order to assess forecast performance. The first work studying the rationality of the forecasts in the SPF was Zarnowitz (1984, 1985) who finds inflation forecast bias, and then a lack of rationality, for the sample period 1968-1979. Keane (1989) studied if professional economists generate rational forecasts using the information of the SPF on the GNP deflator. He found that these forecasts were rational. Using the SPF, Baghestani (2011) studied the rationality of the SPF on the growth rate of the US business investment, proving for this variable and for some horizons that the SPF forecasts are rational, but for others are not.

These empirical studies used the Mincer and Zarnowitz (1969) and Mankiw and Shapiro (1986) tests of unbiasedness. The Mincer-Zarnowitz approach tests whether forecasts are statistically

unbiased whereas the Mankiw-Shapiro approach runs a test for analyzing whether forecast errors are white noise errors. Both tests analyze the unbiasedness of past forecasts.

Focusing on the persistence of the forecast errors, Croushore (2008) used the Livingston Survey<sup>2</sup> and the SPF for analyzing inflation forecasts. He found that forecast error persistence is not stable over time. Thus, the autocorrelation structure of inflation forecast errors does not help for improving inflation forecasts.

Croushore and Marsten (2015) studied whether GPD forecast respond efficiently to monetary policy changes measured by changes in the interest rate yield, using the data base of SPF. However, they do not analyze the properties of interest rate forecasts.

To the best of our knowledge, only Baghestani (2005) has analyzed SPF forecast on interest rates. He showed that the SPF does not provide rational forecasts for the 10-year Treasury bond yield. However, his study showed that the SPF is useful for trend forecasting. His study did not focus on the short-term federal interest rate (3-month Treasury bill) also considered in this master thesis.

### **3. Data**

We have studied the SPF forecasts of two interest rates: the 3-month Treasury Bill yield and the 10-year Treasury Bond yield. The starting sample period of these two yields are 1981:3 and 1992:1, respectively. The actual data is collected from the Federal Reserve Bank of Saint Louis' website<sup>3</sup> whereas the SPF data was downloaded from the website of the Federal Reserve Bank of Philadelphia.<sup>4</sup> In the Philadelphia Fed website, we found all the documentation about the SPF. The profile of a professional forecaster is an economist, forecasting specialist, whose forecasts are used by their companies in the decision making. Once the professional forecasters receive the forecast form, they have to fill a table with their prediction of the quarter-mean for the next 6-quarter horizons and for the next 4-annual horizons (for some variables, there is only 2-annual horizon predictions). Professional forecasters fill out their surveys once the first quarter they have to forecast has started and hence, they have access to the value of the forecasted variable in the previous period. Table 3.1 shows an example of a table that a professional forecaster has to fill out.

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<sup>2</sup> The Livingston Survey was started in 1946 by the late columnist Joseph Livingston. It is the oldest continuous survey of economists' expectations. It summarizes the forecasts of economists from industry, government, banking and academia. The Federal Reserve Bank of Philadelphia took the responsibility for the survey in 1990.

<sup>3</sup> <https://www.stlouisfed.org/>

<sup>4</sup> <http://www.phil.frb.org/>

Table 3.1, example of fill table SPF

	L / G	Quarterly Data					Annual Data <sup>a</sup>					
		2013:Q4	2014:Q1	2014:Q2	2014:Q3	2014:Q4	2015:Q1	2013	2014	2015	2016	2017
1. Nominal GDP		17102.5						16802.9				
2. GDP Price Index (Chain)		107.02						106.47				
3. Corporate Prof After Tax		.						.				
4. Civilian Unemp Rate	L	7.0						7.4				
5. Nonfarm Payroll Employment <sup>b</sup>		136747						135927				
6. Industrial Prod Index		101.2						99.6				
7. Housing Starts		1.002						0.928				
8. T-Bill Rate, 3-month	L	0.06						0.06				
9. AAA Corp Bond Yield	L	4.59						4.24				
10. BAA Corp Bond Yield	L	5.36						5.10				
11. Treasury Bond Rate, 10-year	L	2.75						2.35				

In our analysis we use three type of times series: (i) the actual data f-quarters ahead ( $A_{t+f}$ ), (ii) a naïve forecast based on the most recent observation of the forecasted variable ( $A_t$ ) taking from the data base of the Federal Reserve Bank of Saint Louis, and (iii) the mean forecast of the variable f-quarters ahead from the SPF ( $P_{t+f}$ ) obtained from the Federal Reserve Bank of Philadelphia website. Each of them refers to the mean value of a given quarter (e.g. the mean of the interest rate in the secondary market of the T-Bill for the quarter 2011:4).

## 4. Dimensions of the study

This thesis studies three dimensions of the SPF. Subsection 4.1 explains the dimension of comparison with a naïve forecast, Subsection 4.2 studies the rationality of the forecast and Subsection 4.3 analyzes the trend forecast.

### 4.1 A comparison with a naïve forecast

The naïve forecast is given by the most recent observed value of the forecasted variable (i.e. the forecast of  $A_{t+f}$  is  $A_t$ ). It is one of the simplest forecasts that a forecaster could do. It gives a benchmark for the comparison of the SPF forecasts. Then, the question is: have the mean of SPF provided a better forecast than this naïve forecast?

We first analyze graphically the movement of the interest rate and both forecast, looking for patterns that they have followed. We analyze the errors, calculated as the difference between the actual value and its forecast ( $e_{t+f}=A_{t+f}-P_{t+f}$  and  $e_{t+f}=A_{t+f}-A_t$ ). In addition, the forecast errors are calculated in absolute value. Considering the forecast errors in absolute value looks more appropriate because in this way positive errors do not compensate the negative ones. Moreover, we show the evolution of the accumulated absolute value errors for all the forecast horizons.

### 4.2 Rationality

A rational forecast fulfills two basic properties: <sup>5</sup> (i) unbiasedness and (ii) forecast errors are

<sup>5</sup> Rationality can be called efficiency. This nomenclature is using more often by financial researches.

orthogonal to any variable included in the forecaster information set at the time the forecast was made.

Different authors define with their own words what a rational forecast means:

“We say a forecast is rational if its forecast error is unpredictable, given what the forecaster knew when making the forecast,..., In a strict sense, rationality requires that the forecaster's subjective probability distribution be the same as the objective probability distribution of the forecasted variable. Thus, for a forecast to be rational, it is necessary but not sufficient that forecast errors be unpredictable.” Keane (1989).

“A forecast is rational if it is unbiased and contains all the relevant information publically available at the time of the forecast” Baghestani (2011).

Keane (1989) also focused in why being rational is important for the forecast:

*“For at least sixty years, economists have been concerned with how people forecast economic events. Many economists believe that people make the best economic forecasts they can, given the information available to them at the time. This assumption, called the rational expectations hypothesis; is of more than academic interest because it may imply that some government policies have little effect on people's behavior and therefore on the economy”*

We first analyze the unbiasedness property of rational forecast. All papers reviewed above have studied this condition. There are two ways of making this test: The first one (Mincer-Zarnowitz (1969)) is to regress a model - (1) and (2) - where the actual data is related to a constant term, the forecasted value and an error term behaving as a white noise. If we cannot reject the null hypothesis that  $\alpha=0$  and  $\beta=1$  (alternatively,  $\delta=0$  and  $\lambda=1$  for the naïve forecast), the forecast is considered unbiased and hence, it would fulfill the necessary condition of unbiasedness. The second way (Mankiw-Shapiro (1986)) is to estimate a model - (3) and (4) -<sup>6</sup> with the forecast error as the main explanatory variable together with a constant and error term behaving as a white noise. If we cannot reject the individual null hypotheses that the constant is equal zero ( $\alpha' = 0$  for the SPF and  $\delta'=0$  for the naïve forecasts), we cannot reject that the forecasts have been unbiased for that horizon.

$$A_{t+f} = \alpha + \beta P_t + u_{t+f} \quad (1)$$

$$A_{t+f} = \delta + \lambda A_t + z_{t+f} \quad (2)$$

$$A_{t+f} - P_t = \alpha' + e_{t+f} \quad (3)$$

$$A_{t+f} - A_t = \delta' + w_{t+f} \quad (4)$$

The next step, after having studied the unbiasedness of the forecast, is to study the persistence of the residuals from these regressions. This is a necessary condition for being

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<sup>6</sup> Note: Using the residuals of the regressions of these two models we will study the persistence in the forecasts.

rational because if the estimated residuals have structure means they have not behaved as a white noise and they could have been predictable. Then, we would have to reject the null hypothesis of rationality. We use the residuals from the estimation of models (3) and (4) for discovering if they show an autocorrelation pattern. We run autoregressions as (5) and (6) to test whether residuals are autocorrelated for each horizon:

$$\hat{e}_{t+f} = c + \varphi_1 \hat{e}_{t+f-1} + \dots + \varphi_n \hat{e}_{t+f-n} + v_{t+f} \quad (5)$$

$$\hat{w}_{t+f} = c + \sigma_1 \hat{w}_{t+f-1} + \dots + \sigma_n \hat{w}_{t+f-n} + x_{t+f} \quad (6)$$

If all coefficients associated with lagged values of residuals are zero, we cannot reject the null hypothesis, which is a necessary condition for rational forecasting.

### 4.3 Trend forecast

One of the most important forecasts is the trend forecast. That is, the forecast about the direction of the interest rate in each horizon. This analysis helps us to understand how accurate professional forecasters have forecasted the trend. By definition, the naïve forecast implies that the variable is not going to move. Thus, it does not make any change in trend forecast.

We measure the trend forecast as follows. We compute the difference between the mean of the SPF forecasts for a particular horizon ( $P_{t+f}$ ) and the last value of reference ( $A_t$ ). If this difference is positive means forecasters are predicting a more probable uptrend from the quarter  $t$  to quarter  $f$ ; the opposite for a downtrend. The first approximation is to count how many times they have well guessed the direction of interest rates. Giving a reference of the probability they have had of forecasting well the direction.

This approach is not enough for describing how good the trend forecast of SPF is. We have created a scoring method for evaluating every trend forecast for every quarter. It approximates to the results of decisions taking with the information of the direction giving by the mean of SPF. For example, investing in future markets where you can situate long, short or out of the market. This consists once the mean of SPF forecasts a more probable uptrend, the scoring method gives to this forecast a value – being the unit basis point (1%)- equivalent to the difference between the actual value in the forecast horizon ( $A_{t+f}$ ) and the last available value ( $A_t$ ) as in (7). If the forecast is downtrend the difference is computed as in (8). Alternatively, the scoring method uses as reference the next value after forecasting ( $A_{t+1}$ ), because in a practical field the available data has to be post the decision.

$$\text{If } P_{t+f} > A_t \Rightarrow \text{forecast } \uparrow \Rightarrow \pi_{t+f} = A_{t+f} - A_t \text{ or } \pi'_{t+f} = A_{t+f} - A_{t+1} \quad (7)$$

$$\text{If } A_t > P_{t+f} \Rightarrow \text{forecast } \downarrow \Rightarrow \pi_{t+f} = A_t - A_{t+f} \text{ or } \pi'_{t+f} = A_{t+1} - A_{t+f} \quad (8)$$

Therefore we get the  $\pi_{t+1}$  and  $\pi'_{t+1}$  and it gives the possibility of studying the evolution of the accumulated score. Their evolution shows us if they produced good trend forecasts. A positive slope and low dispersion (Maximum drawdown<sup>7</sup>) indicates that the mean of the SPF have done a good job in forecasting the trend in the past and there will be more possibilities of doing in the future.

## 5. An analysis of the 3-month Treasury Bill forecasts

This section answers the three questions posited in this thesis for the 3-month T-Bill yield (Period 1981:3 – 2015:1). Subsection 5.1 compares the SPF forecast with the naïve forecast. Subsection 5.2 analyzes rationality. Finally, Subsection 5.3 studies the trend forecast.

### 5.1 A comparison with a naïve forecast, T-Bill

The first step for studying the SPF forecast for the 3-month Treasury Bill includes a comparison with the naïve forecast. As explained before, the naïve forecast is just given by the most recent observed value of the forecasted variable (i.e. the forecast of  $A_{t+1}$  is  $A_t$ ). Table A.1 of Appendix 1 shows three time series: the actual data of the interest rate, the mean forecast of the SPF and the naïve forecast for every forecast horizon. Graphically, the values predicted by the SPF follow closely the actual values with a lag. The SPF forecasts followed the trend of the actual values and once the trend changes, professional forecasters seem to react with a lag to the observed switching direction. The same pattern occurs with the naïve forecast. These graphs show that forecast errors are rather persistent.

Comparing the results of the forecast errors associated with the SPF and the Naïve forecast (Table A.2 in Appendix 1), the mean of the errors is lower for the Naïve forecast than for the SPF, but the standard deviation is greater for all the horizons forecasted.

The next approximation is to carry out the statistical analysis in absolute value of forecast errors. Results show that the mean and the standard deviation for the SPF are lower than for the naïve forecast (Table A.3 in appendix 1) except for the shortest forecast horizon.

Table A.4 in appendix 1 shows the accumulated errors in absolute value for the SPF and Naïve forecasts. The accumulated errors, in absolute value, have been lower for the SPF forecast than for the naïve forecast for all forecast horizons.

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<sup>7</sup> Note: Drawdown measures the maximum distance between the historical highest peak and the actual value for each period. Once we have calculated this series, we look for the maximum value of it. This gives us a measure called Maximum drawdown which let us to know the maximum negative accumulated fall of the accumulated score. This statistic is quite often used for analyzing algorithm decision in financial markets, which gives the maximum lost one investor would have accumulate if she/he would have used this decision system in the past. It is a good measure of dispersion of a series and also a good measure of the maximum error accumulated taken if one use a decision rule: in our case, the difference between the mean of the SPF and the last available data. [https://en.wikipedia.org/wiki/Drawdown\\_\(economics\)](https://en.wikipedia.org/wiki/Drawdown_(economics))

In conclusion, the two forecast errors for the 3-month Treasury Bill yield show persistence patterns. Moreover, the SPF forecasts are better than the naïve ones when focusing on absolute value of prediction errors for all horizons.

## 5.2 Rationality, T-Bill

Table A.5 in appendix 1 corresponds to the regression of the model (1) and model (3) for the SPF forecasts of the short term federal interest rate. Test statistics for knowing if we cannot reject the null hypothesis that  $\alpha=0$  and  $\beta=1$  for model (1) and  $\alpha' = 0$  for model (3) are summarize in Table 5.2.1. We reject the null hypothesis of the join test that the forecast of the SPF have been unbiased for all horizons at the 95% confidence level. Regarding test (3), we reach to the same conclusion. We can reject the null hypothesis of rationality at the 95% significance level.

Table 5.2.1 p-values of the regressions (1) and (3) for all forecast horizons, T-Bill

Regression (1)	1	2	3	4	5	6
p-value F-test ( $\alpha=0; \beta=1$ )	0.001	0.000	0.000	0.000	0.000	0.000
Regression (3)						
p-value ( $\alpha'=0$ )	0.025	0.003	0	0	0	0

For the naïve forecasts, the results are in Table A.6 in appendix 1 corresponding to the models (2) and (4) and the summary is shown in Table 5.2.2. If we cannot reject the null hypothesis of the joint test  $\delta=0$  and  $\lambda=1$  for model (2) and the individual test  $\delta'=0$  for model (4) at the 95% level of confidence means that the hypothesis of forecast unbiasedness is not rejected by the data. In this case the values of the tests show that we can reject the null hypothesis and therefore the SPF forecasts have not been unbiased at the 95% level of confidence for all horizons.

Table 5.2.2 p-values regressions (2) and (4) for all forecast horizons

Regression (2)	1	2	3	4	5	6
P-value F-test ( $\delta=0; \lambda=1$ )	0.001	0.000	0.000	0.000	0.000	0.000
Regression (4)						
p-value ( $\delta'=0$ )	0.034	0.008	0.004	0.002	0.001	0.001

Table 5.2.3<sup>8</sup> summarizes the analysis of residual persistence for each forecast horizon of the SPF and naïve forecasts for the short-term interest rate. It shows the autocorrelation order for each forecast horizon. Regarding SPF forecasts, residual persistence is rather stable across forecast horizons. In particular, we found there is a significant autocorrelation of order 1 for the first two horizons and of order 2 for the horizons 3, 4, 5 and 6 quarters. For Naïve forecasts, the autocorrelation order is much more unstable, for one-quarter horizon the autocorrelation is of order 1, for 2-quarter horizons autocorrelation is of order 4, for horizons 3, 5 and 6 the autocorrelation are of order 2 and for 4-quarter horizon it is of order 3. These results suggest that the forecast errors for SPF and naïve forecasts for all horizons exhibit persistence and, then, they display certain degree of predictability. In sum, the two type of forecast (SPF and naïve) do not fulfilled this necessary condition for rationality.

Table 5.2.3 Residuals autocorrelation order for all the horizons (95% confidence level)

<b>Autocorrelation order</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>
<b>SPF</b>	1	1	2	2	2	2
<b>Naive</b>	1	4	2	3	2	2

This subsection has shown that SPF forecasts for the quarter mean of the 3-month Treasury Bill yield cannot be considerate rational, because they have not fulfilled each of the two necessary conditions for being rational. We have rejected the hypothesis of unbiasedness for SPF forecast for all horizons. Moreover, we have found that regression residual as well as estimated forecasted errors show persistence patterns. For the Naïve forecasts the null hypothesis of unbiasedness is rejected for all horizons and the estimated residuals as well as the estimated forecast errors have not behaved as white noise processes.

### 5.3 Trend forecast, T-Bill

Analyzing the percentage rate of guessing the true direction of a change, the SPF forecasts were right more than 57% of the times for all horizons, but the shortest one, where they got it right a 46.67%. We highlight the 73.88% accuracy in the trend forecast for the 2-quarter horizon. These results suggest that the SPF forecast does much better than a randomly trend forecast.

<sup>8</sup> Note: All regression results can be found in Tables A.7 and A.8, appendix 1.

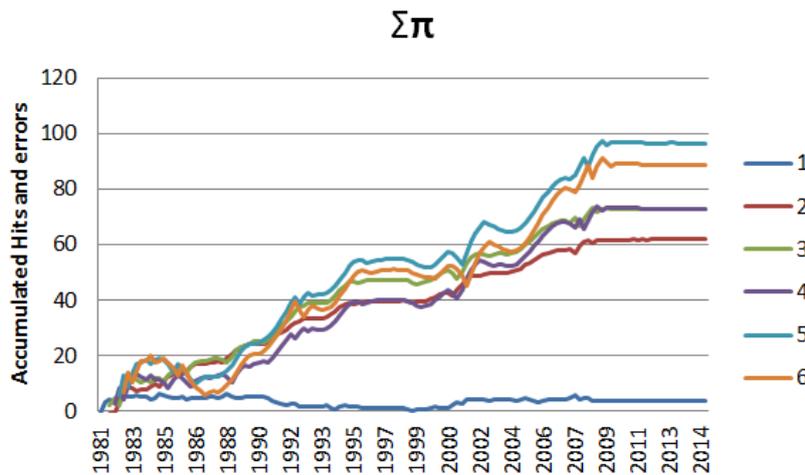
Table 5.3.1 Percentage rate of guessing the true direction of a change for alternative forecast horizons, T-Bill

Accuracy	1	2	3	4	5	6
Observations	135	134	133	132	131	130
% right	46.67	73.88	65.41	58.33	60.31	57.69

For analyzing the performance of the SPF over time, we have two graphs with the accumulated score of each horizon forecasts. The first graph  $\Sigma\pi$  shows the evolution of the score for the trend forecast from the first quarter (3<sup>rd</sup> quarter of 1981) to the last one (1<sup>st</sup> quarter 2015) for all forecast horizons, using the reference value  $A_t$ . The graph  $\Sigma\pi'$  is similarly computed, but using as the reference value for scoring  $A_{t+1}$ .

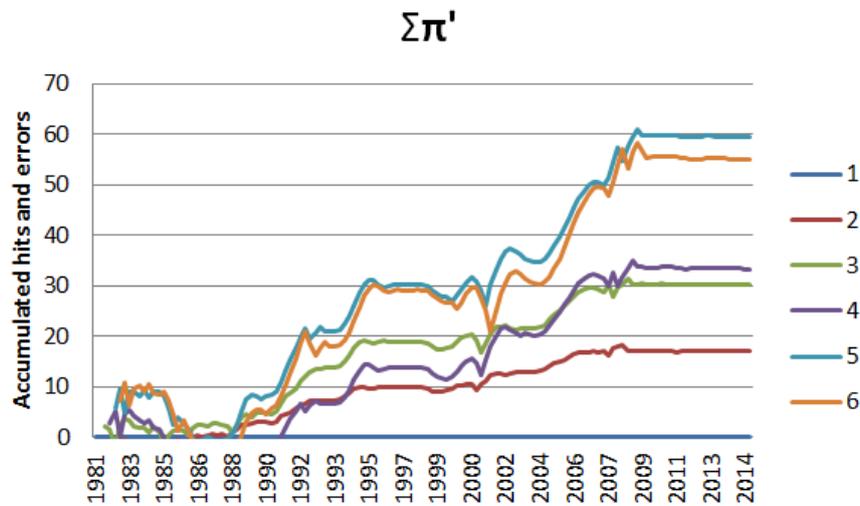
Using as reference  $A_t$ , Table A.9 appendix 1 shows that the accumulated score have moved with uptrend for all forecast horizons. The best forecast is the 5-quarter horizon, but results also suggest that longer horizons are associated with more dispersion: both standard deviations and drawdown statistics have increased with longer forecast horizons.

Graph 5.3.2 Accumulated score  $\pi$  for all forecast horizons, T-Bill



When considering  $A_{t+1}$  as a reference, results change. Thus, all accumulated scores are lower. Using  $\Sigma\pi$  the total accumulated scores moved between 60 and 100. However, using  $\Sigma\pi'$  total accumulated scores are between 15 and 60. Analyzing the statistics summarized in Table A.10 in appendix 1 the total accumulated scores decrease and the standard deviations and drawdowns increase. Using the accumulated  $\pi'$ , the best performance is obtained with the 5-quarter horizon as occurred with the accumulated  $\pi$  and the slope is clearly positive for all horizons, although the scores do not show a positive slope from the eighties to the early nineties.

Graph 5.3.3 Accumulated score  $\pi'$  for all the forecast horizons, T-Bill



In sum, the SPF does a good job in capturing the trend of the US short-term interest rate. Moreover, the best performance is obtained for the 5-quarter horizons according to our scoring methods. In particular, the scores exhibit positive slopes for all forecast horizons, except the shortest one. Furthermore, the scores show higher dispersion for longer forecast horizons: greater standard deviations and drawdowns.

## 6. An analysis of the 10-year Treasury Bond forecasts

This section carries out the same steps implemented in the previous section in order to study the forecast associated with the 10-year Treasury Bond yield (i.e. a long-term interest rate) forecasts. Data for these forecasts are only available from the first quarter of 1992, which implies a much shorter sampling period than the one available for the short-term interest rate forecasts.

### 6.1 Comparison with a naïve forecast, T-Bond

The graphs of the three series are in the table A.11 appendix 2: Actual data, naïve forecasts and the mean of the SPF forecasts. The three shortest forecast horizons have followed a visual pattern parallel to the actual data, needing time for adapting to a change in the trend; nevertheless with longer horizons this pattern starts to disappear and they have moved more randomly around the actual data losing little by little this pattern. Other highlight of the graphical analysis is errors  $(A_{t+f} - P_t)$  seem more often negative than positive in the series.

Reviewing the statistical analysis in the table A.12 appendix 2 for the errors we find the mean for the naïve forecast is lower but the dispersion is higher, this happened for the T-bill forecasts too.

In the table A.13 appendix II analyzing the mean of errors in absolute value, the first three horizons the SPF forecasts have been better than the naïve one but with longer horizons naïve forecasts have become a better way of forecasting. In table A.14 appendix 2 the accumulated error series for all the horizons. It is visually clear that for a horizon of two quarters the SPF forecast has behaved better than the naïve in absolute value errors. For horizons of 1, 3 and 4 have had similar behaviors and for horizons of 5 and 6 quarters accumulate errors have increased slower in the naïve forecasts than in the SPF, especially in the 25 last quarters.

## 6.2 Rationality, T- Bond

Repeating the same analysis made it in point 5.2, we use models (1), (2), (3) and (4) for testing the null hypothesis of unbiasedness in the forecast for the 6 horizons.

Analyzing (1) test for SPF forecasts on T-Bond, we reject the null hypothesis that  $\alpha=0$  and  $\beta=1$  at 95% level of confidence for all the forecast horizons except for the shortest one. It means we cannot reject the null hypothesis of unbiasedness using this test model for 1 quarter horizon. Focusing in the test (3) ( $\alpha'=0$ ), the conclusions are the same. We reject the null hypothesis of unbiasedness for all the horizons except the shortest at 95% level of confidence. In table A.16 appendix 2 we have the values of the regressions for these two tests.

Table 6.2.1 p-values regressions (1) and (3) for all forecast horizons, T-Bond

<b>Regression (1)</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>
<b>p-value F-test (<math>\alpha=0</math>; <math>\beta=1</math>)</b>	0.186	0.019	0.000	0.000	0.000	0.000
<b>Regression (3)</b>						
<b>p-value (<math>\alpha'=0</math>)</b>	0.149	0.011	0	0	0	0

Reviewing the unbiasedness tests for naïve forecast for T-Bond and using tests (3) and (4) (Table 6.2.2), the results are equivalent of SPF. We have rejected the null hypothesis of unbiasedness for both tests in 2, 3, 4, 5 and 6 quarters forecast horizons. For the shortest forecast horizon we cannot reject the null hypothesis of unbiasedness at 95% level of confidence. In table A.17 appendix 2 are the full values of the estimations.

Table 6.2.2 p-values regressions (2) and (4) for all forecast horizons, T-Bond

<b>Regression (2)</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>
<b>F-test (<math>\alpha=0</math>; <math>\beta=1</math>)</b>	0.184	0.035	0.005	0.002	0.000	0.000
<b>Regression (4)</b>						
<b>p-value (<math>\delta'=0</math>)</b>	0.146	0.069	0.028	0.016	0.007	0.002

The second necessary condition is about the persistence of the forecasts errors. It means if residuals have behaved as a white noise or not. Following the same test we did for the

forecasts for the 3 months Treasury Bill we have gotten the autocorrelation of the residuals of models (5) and (6) for SPF and Naïve forecasts. The results of the estimation of the models are in tables A.17 and A.18 in appendix 2. At table 6.2.3<sup>9</sup> the summary of results at 95% level of confidence is telling the autocorrelation of the residuals. Result for a horizon of 1 quarter says they behaved as a white noise, without any statistical autocorrelation in the residuals. At longer horizons for both forecasts we have found statistical autocorrelation of order 2. It means SPF and naïve forecasts have fulfilled the necessary conditions of no persistence in the horizon of 1 quarter, for longer horizons, they have not fulfilled the necessary conditions of not persistence.

Table 6.2.3 Residuals autocorrelation order for all the horizons (95% confidence level)

<b>Autocorrelation order</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>
<b>SPF</b>	0	2	2	2	2	2
<b>Naïve</b>	0	2	2	2	2	2

After analyzing these two necessary conditions for both kind of forecasts for the ten years Treasury Bond the results shows for an horizon of 1 quarter we cannot reject the hypothesis of rationality in the forecasts at 95% level of confidence. For longer horizons we reject the hypothesis of rationality at 95% level of confidence for SPF and naïve forecasts because they have not fulfilled the necessary conditions of unbiasedness and persistence in the errors.

### 6.3 Trend forecast, T-Bond

The last part of the error forecasts analysis is to study their historical trend forecasts. Following the same steps we did in the point 5.3, we find the percentage of the time that the mean of the SPF was right indicating the trend was low for all the horizons except for 2 quarters. It is especially low for the horizon of one quarter, with only 37.63 of right guess. For the horizon of two quarters it was right in the 65.22% but for the rest of longer horizon they are situated between 47.78% and 54.95%. It means they have not been right in their mean intuition in the trend of long term interest rate, except for horizon of 2 quarters.

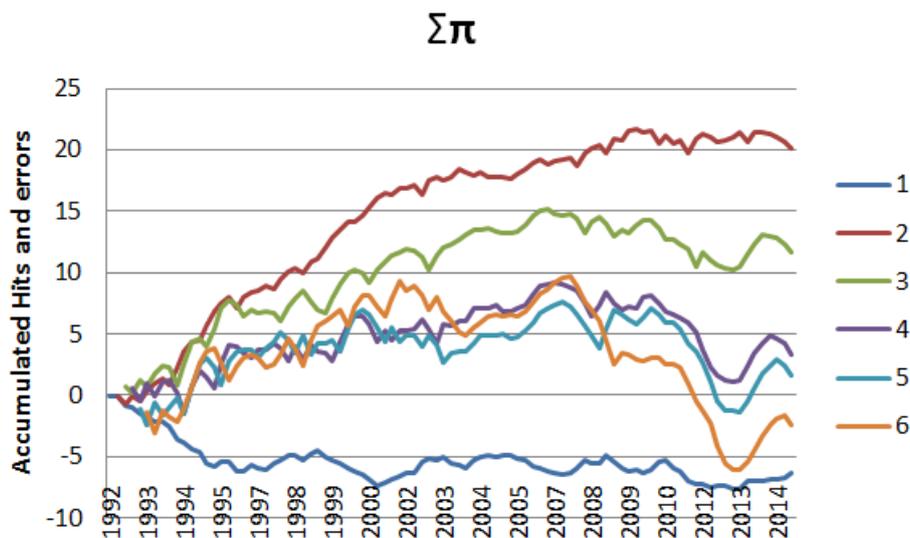
Table 6.3.1 well guess direction for all the forecast horizon, T-Bond

<b>Accuracy</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>
<b>Observations</b>	93	92	91	90	89	88
<b>% right</b>	37.63	65.22	54.95	47.78	50.56	51.14

<sup>9</sup> The full results of the estimation are in tables A.17 and A.18 in an appendix 2

Remembering, we have designed a scoring method for analyzing the effectiveness of these trend opinions got from the difference between the mean of the SPF forecasts and the last available actual value ( $A_t$ ). As it was done in the graph 5.3.2, in graph 6.3.2 it is the accumulated score for the approximation having as reference  $A_t$ . In table 19 appendix 2 we have all the statistical analysis of these series. Graphically the behavior of forecasts for 1 quarter horizon is close to zero and the maximum accumulated score is 20.17 basis points for the horizon of 2 quarters. In opposite what we saw with the short term interest rate SPF trend forecasts, as long as the forecast horizon is, worse the accumulated score of the trend forecasts are, being even total accumulated score negative for horizon of 6 quarters. Moreover the maximum drawdown, which is our statistic of dispersion, show in longer horizons these are higher, therefore the accumulated maximum loss of score is worse. Reviewing the coefficients of linear estimation of the accumulated scores we find that only for horizons of 2, 3 and 4 quarters these are significant positive at 95% level of confidence. After these results we have only found as good trend forecasts horizons 2 and 3 quarters if we have as reference of this scoring method  $A_t$ .

Graph 6.3.2 accumulated score  $\pi$  for all the forecast horizons, T-Bond

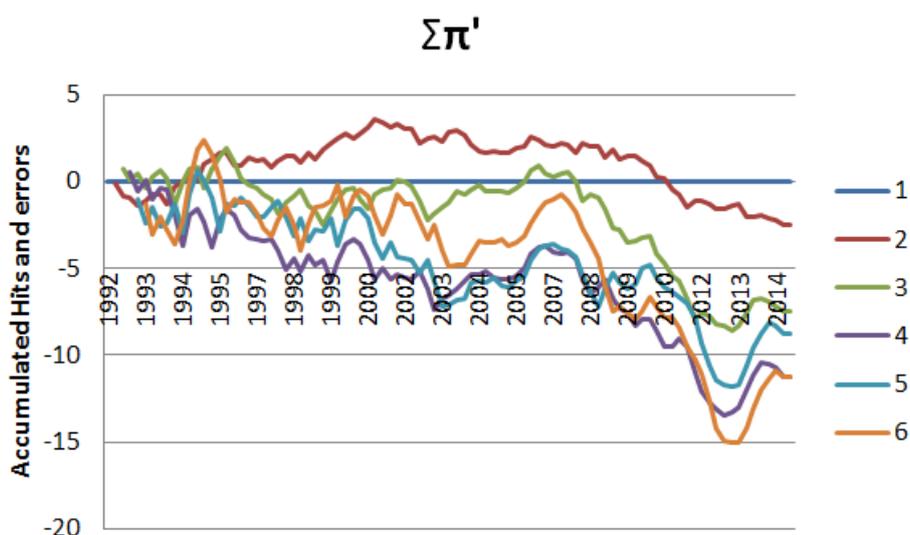


Continue with the trend forecast analysis we have to make the same analysis, as we explained in 4.3, taking as reference value  $A_{t+1}$ . At this case we use for calculate the score of every trend forecasts is the difference between  $A_{t+f}$  and  $A_{t+1}$ . In the graph 6.3.3 (The statistical analysis is in table 20 appendix 2) we have the accumulated score series for every horizon forecasts. The first conclusion is that all total accumulated scores for all the forecast horizons are negative<sup>10</sup>, between -2.47 and -11.28. Also analyzing the results of the estimation of a lineal model all the coefficients are negative significant at 95% level of confidence and about the dispersion, using the Maximum drawdown, we find with longer forecasts horizons this statistic increases, except

<sup>10</sup> The score for the horizon of 1 quarter is 0 because  $A_{t+1}=A_{t+f}$

for the accumulated score of 4 quarter horizon which is higher than for forecasts in 5 horizons. Bearing in mind all of these results, having as reference  $A_{t+1}$  for scoring the trend forecasts of the mean SPF, we have found they have not been historical worth for forecasting the trend of the ten years Treasury Bond in any horizon.

Graph 5.3.3 accumulated score  $\pi'$  for all the forecast horizons, T-Bill



In conclusion, the mean of SPF have forecasted badly the trend of the long term Treasury Bond following our scoring method. This results is opposite to what Baghestani (2005) found.

## 7. Conclusions

This master thesis studied the properties of the mean forecasts of the SPF associated with the US short- and the long-term interest rates. We answered three questions:

- Have professional forecasters made a better forecast than a naïve forecaster?
- Have professional forecasters been rational?
- Have professional forecasters made good trend direction forecasts of interest rates?

For the short-term interest rate, we found that the mean of the SPF have been a better way of forecasting than a naïve forecast based on the last observed interest rate value. Moreover, standard tests reject the hypothesis of rationality for the two forecast (SPF and naïve). Furthermore, the mean of the SPF does a good job in capturing the upward or downward trends of the short-term interest rate according to the scoring methods considered in this research.

For the T-bond, results are very different. In particular, naïve forecast perform better than the mean of the SPF forecast for forecast horizons longer than 3 quarters. Moreover, for the shortest forecast horizon, we cannot reject the hypothesis of forecast rationality neither for the mean of the SPF nor for the naïve forecasts. For the rest of forecast horizons, we found

that the two forecasts do not fulfill the two necessary conditions for rationality. Regarding trend prediction, our empirical evidence suggests, according to our scoring methods, the mean of the SPF does not provide good trend forecasts.

Table 7.1 Summary of the three dimensions studied across the two variables

Dimension	T-Bill	T-Bond
Comparison with a naïve forecast	The mean of SPF has been a better way of forecasting than a naïve model.	The mean of SPF has been a better way of forecasting than a naïve model only for forecast horizons of 1, 2 and 3 quarters.
Rationality	The mean of SPF and a naïve model have not fulfilled the 2 necessary conditions for rationality. The hypothesis of rationality is rejected for all forecast horizons.	For the shortest forecast horizon, the naïve forecast and the mean of SPF have behaved unbiased and with no persistence in forecast errors. Hence, we cannot reject the null hypothesis of rationality. For longer horizons, we have rejected these two necessary conditions.
Trend Forecast	Based in our scoring method, would have been useful to use the information given by the mean of SPF for forecasting the trend of the 3-month interest yield.	If we concentrate in our scoring method would not have been worth to use the information given by the mean of SPF for forecasting the direction the 10-year Treasury Bond.

We conclude the mean of the SPF have forecasted well the US short-term interest rate, but they have not done well in forecasting the long-term interest rate. A potential explanation of these different results is that the 3-month interest rate is much easier to forecast than long-term interest rates, as the 10-year yield, because short-term interest rate are much determined by monetary policy which has a strong inertial component much easier to predict.

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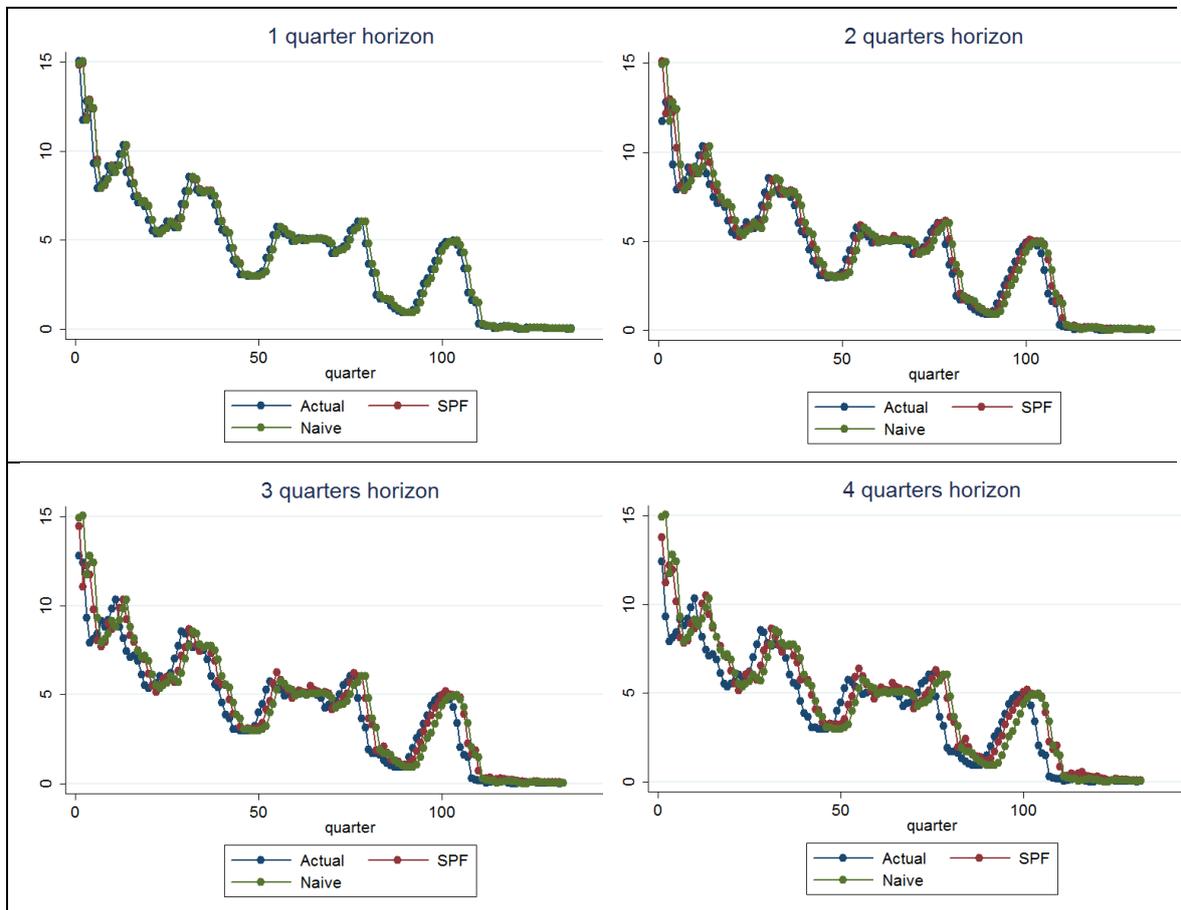
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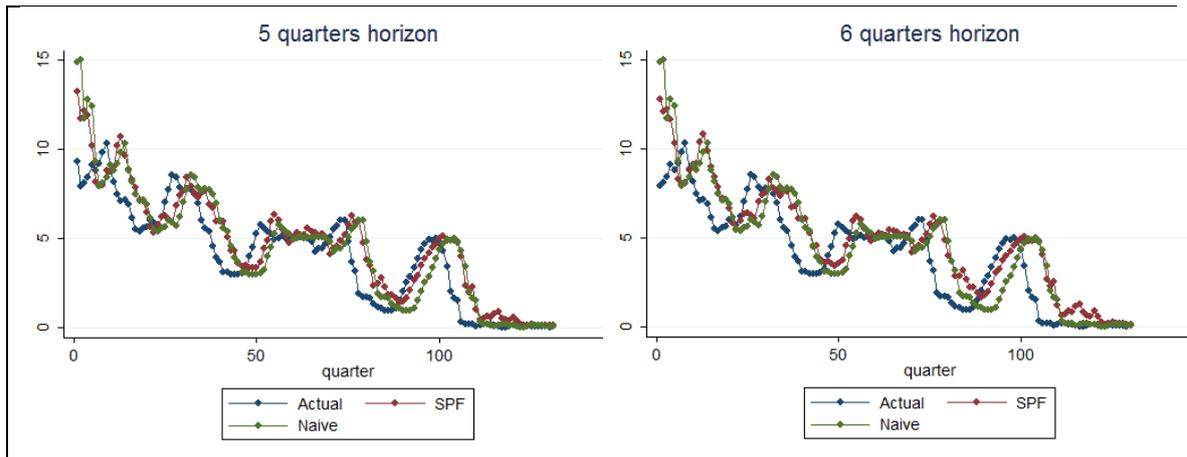
## Appendix 1

In annex I we find all the tables and graphs related with the analysis of the error forecasts of three months Treasury Bill.

**Table A.1**

These are graphs of the comparison of the actual value ( $A_{t+f}$ ), the mean of SPF ( $P_{t+f}$ ) and the naïve forecast ( $A_t$ ) for the 3 months Treasury Bill for every horizon quarterly.





**Table A.2**

It is a statistical description of the forecast error for the survey professional of forecasters and the naïve forecast for the short term Treasury Bill for every horizon.

SPF	1	2	3	4	5	6
<b>Observations</b>	135	134	133	132	131	130
<b>Mean SPF error</b>	-0.116	-0.178	-0.289	-0.469	-0.662	-0.846
<b>Std. Dev. SPF error</b>	0.594	0.674	0.911	1.163	1.413	1.594
<b>Min SPF error</b>	-3.185	-3.353	-3.843	-4.299	-4.054	-4.872
<b>Max SPF error</b>	0.890	1.081	1.468	1.989	2.646	2.327
<b>Naïve</b>						
<b>Mean Naive error</b>	-0.110	-0.223	-0.313	-0.412	-0.510	-0.585
<b>Std. Dev. Naive error</b>	0.598	0.963	1.232	1.528	1.795	1.986
<b>Min Naive error</b>	-3.300	-4.510	-4.900	-5.730	-7.140	-7.000
<b>Max Naive error</b>	1.060	1.530	2.330	2.820	2.690	2.900



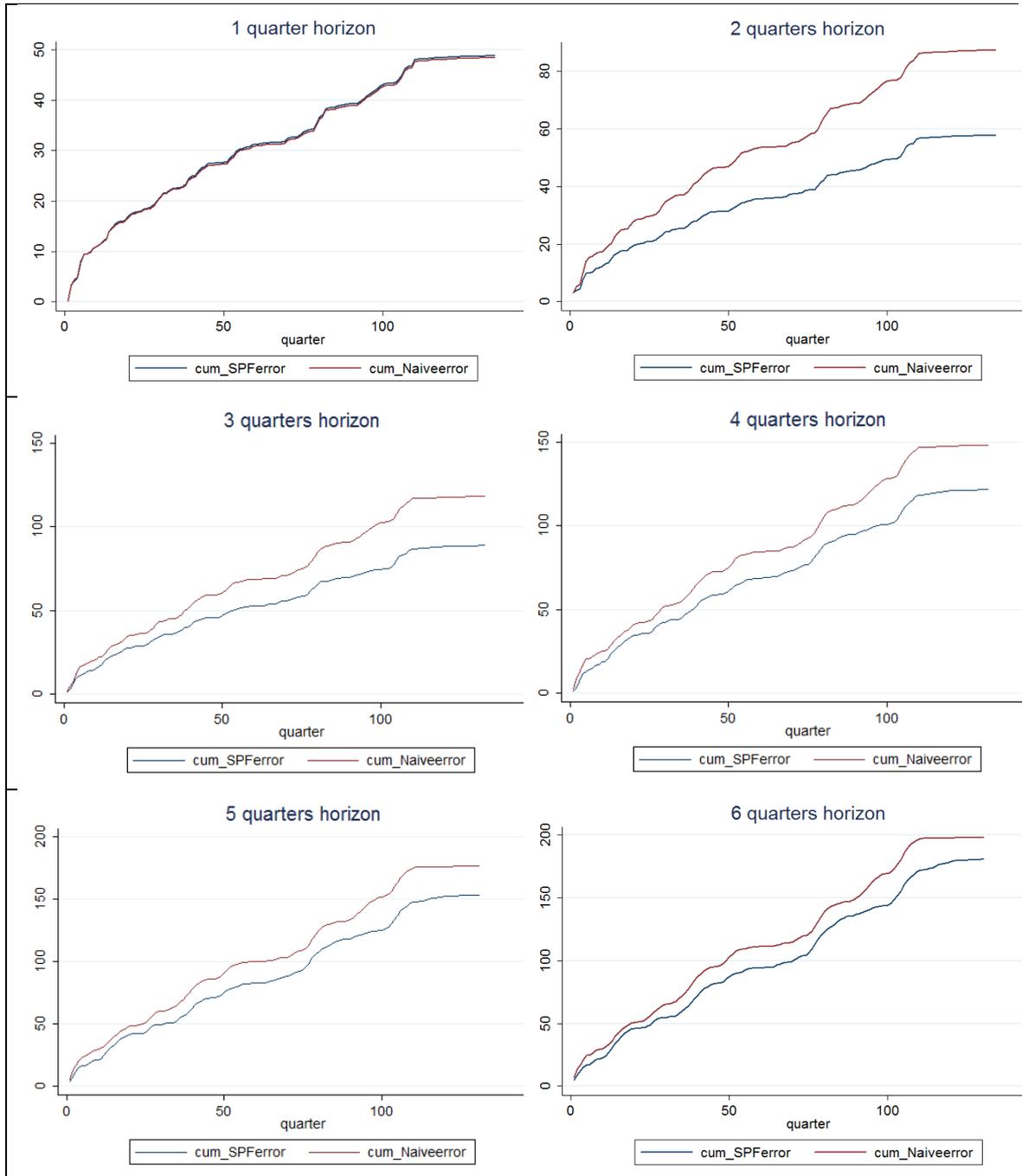
**Table A.3**

It is a statistical description of the forecast errors in absolute values for the survey professional of forecasters and the naïve forecast for the short term Treasury Bill.

<b>SPF</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>
<b>Observations</b>	135	134	133	132	131	130
<b>Mean ABS SPF error</b>	0.361	0.431	0.668	0.922	1.169	1.389
<b>Std. Dev. ABS SPF error</b>	0.485	0.547	0.681	0.848	1.030	1.148
<b>Min ABS SPF error</b>	0.000	0.005	0.001	0.010	0.002	0.022
<b>Max ABS SPF error</b>	3.185	3.353	3.483	4.299	4.054	4.872
<b>Naïve</b>						
<b>Observations</b>	135	134	133	132	131	130
<b>Mean ABS Naïve error</b>	0.359	0.652	0.888	1.121	1.345	1.523
<b>Std. Dev. ABS Naïve error</b>	0.490	0.741	0.907	1.113	1.289	1.396
<b>Min ABS Naïve error</b>	0.000	0.000	0.000	0.000	0.010	0.000
<b>Max ABS Naïve error</b>	3.300	4.510	4.900	5.730	7.140	7.000

**Table A.4**

It is the graphs of accumulated errors in absolute value for SPF and naïve forecasts for the short term interest rate.



**Table A.5**

It is the estimation of model (1) and (3) for forecasts on 3 months Treasury Bill.

Regression (1)	1	2	3	4	5	6
$\alpha$	0.101	0.087	0.019	0.042	0.080	0.029
Std. Dev.	0.083	0.096	0.136	0.176	0.219	0.259
p-value ( $\alpha=0$ )	0.222	0.367	0.889	0.812	0.717	0.912
$\beta$	0.951	0.940	0.931	0.888	0.842	0.819
Std. Dev.	0.015	0.018	0.025	0.032	0.039	0.046
p-value ( $\beta=0$ )	0.000	0.000	0.000	0.000	0.000	0.000
t-value ( $\beta=1$ )	-3.275	-3.409	-2.752	-3.482	-3.994	-3.927
p-value ( $\beta=1$ )	0.189	0.076	0.071	0.025	0.010	0.008
p-value F-test ( $\alpha=0; \beta=1$ )	0.001	0.000	0.000	0.000	0.000	0.000
<b>R<sup>2</sup></b>	<b>0.968</b>	<b>0.956</b>	<b>0.913</b>	<b>0.855</b>	<b>0.779</b>	<b>0.713</b>
Regression (3)						
$\alpha'$	-0.116	-0.178	-0.289	-0.469	-0.662	-0.846
Std. Dev.	0.051	0.058	0.079	0.101	0.123	0.140
p-value ( $\alpha'=0$ )	0.025	0.003	0.000	0.000	0.000	0.000
<b>R<sup>2</sup></b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>	<b>0.000</b>

**Table A.6**

It is the estimation of model (2) and (4) for forecasts on 3 months Treasury Bill.

Regression (2)	1	2	3	4	5	6
$\delta$	0.103	0.260	0.362	0.537	0.729	0.847
Std. Dev.	0.083	0.131	0.167	0.204	0.236	0.261
p-value ( $\delta=0$ )	0.221	0.048	0.032	0.009	0.002	0.001
$\lambda$	0.952	0.891	0.849	0.790	0.728	0.688
Std. Dev.	0.015	0.024	0.030	0.037	0.042	0.047
p-value ( $\lambda=0$ )	0	0	0	0	0	0
t-value ( $\lambda=1$ )	-3.180	-4.595	-5.005	-5.738	-6.452	-6.713
p-value ( $\lambda=1$ )	0.194	0.044	0.015	0.005	0.001	0.001
p-value F-test ( $\delta=0; \lambda=1$ )	0.001	0.000	0.000	0.000	0.000	0.000
<b>R<sup>2</sup></b>	<b>0.967</b>	<b>0.915</b>	<b>0.859</b>	<b>0.781</b>	<b>0.697</b>	<b>0.631</b>
Regression (4)						
$\delta'$	-0.110	-0.223	-0.313	-0.412	-0.510	-0.585
Std. Dev.	0.051	0.083	0.107	0.133	0.157	0.174
p-value ( $\delta'=0$ )	0.034	0.008	0.004	0.002	0.001	0.001
<b>R<sup>2</sup></b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>

**Table A.7**

In these tables we can find the results of the estimation of model (5) for fulfilling the table 5.2.2. The value of “n” indicates the number of lags of model (5).

<b>Regression SPF (n=1)</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>
<b>c</b>	-0.002	0.024	0.011	0.009	0.029	0.036
<b>Std. Dev.</b>	0.049	0.047	0.061	0.057	0.058	0.059
<b>p-value (c=0)</b>	0.966	0.604	0.851	0.869	0.622	0.538
<b>φ1</b>	0.320	0.443	0.638	0.827	0.863	0.882
<b>Std. Dev.</b>	0.082	0.070	0.067	0.049	0.041	0.037
<b>p-value (φ1=0)</b>	0.000	0.000	0.000	0.000	0.000	0.000
<b>R<sup>2</sup></b>	0.103	0.235	0.414	0.687	0.775	0.818
<b>Regression SPF (n=2)</b>						
<b>c</b>	0.022	0.003	-0.010	0.010	0.018	0.015
<b>Std. Dev.</b>	0.043	0.043	0.056	0.049	0.050	0.050
<b>p-value (c=0)</b>	0.614	0.950	0.854	0.841	0.719	0.769
<b>φ1</b>	0.337	0.650	0.827	1.255	1.307	1.381
<b>Std. Dev.</b>	0.076	0.080	0.081	0.075	0.076	0.075
<b>p-value (φ1=0)</b>	0.000	0.000	0.000	0.000	0.000	0.000
<b>φ 2</b>	0.016	-0.127	-0.242	-0.519	-0.500	-0.533
<b>Std. Dev.</b>	0.076	0.073	0.080	0.075	0.075	0.073
<b>p-value (φ 2=0)</b>	0.831	0.085	0.003	0.000	0.000	0.000
<b>R<sup>2</sup></b>	0.147	0.358	0.497	0.772	0.827	0.868
<b>Regression SPF (n=3)</b>						
<b>c</b>	0.003	0.018	1.083	0.030	0.024	0.018
<b>Std. Dev.</b>	0.039	0.040	0.066	0.045	0.050	0.049
<b>p-value (c=0)</b>	0.946	0.659	0.000	0.511	0.629	0.722
<b>φ1</b>	0.532	0.802	-0.607	1.234	1.256	1.293
<b>Std. Dev.</b>	0.078	0.081	0.081	0.082	0.089	0.088
<b>p-value (φ1=0)</b>	0.000	0.000	0.000	0.000	0.000	0.000
<b>φ 2</b>	-0.129	-0.433	0.214	-0.533	-0.391	-0.310
<b>Std. Dev.</b>	0.073	0.091	0.062	0.124	0.139	0.142
<b>p-value (φ 2=0)</b>	0.079	0.000	0.001	0.000	0.006	0.031
<b>φ 3</b>	0.202	0.254	0.028	0.022	-0.085	-0.166
<b>Std. Dev.</b>	0.068	0.068	0.042	0.081	0.087	0.086
<b>p-value (φ 3=0)</b>	0.003	0.000	0.511	0.792	0.326	0.055
<b>R<sup>2</sup></b>	0.318	0.464	0.706	0.791	0.823	0.870

**Table A.8**

In these tables we can find the results of the estimation of model (6) for fulfilling the table 5.2.2.

Regression Naive (n=1)	1	2	3	4	5	6
c	-0.002	0.023	0.016	0.018	0.042	0.054
Std. Dev.	0.050	0.059	0.057	0.064	0.065	0.065
p-value (c=0)	0.974	0.695	0.786	0.775	0.515	0.409
$\sigma_1$	0.288	0.660	0.840	0.870	0.878	0.885
Std. Dev.	0.083	0.061	0.046	0.042	0.036	0.033
p-value ( $\sigma_1=0$ )	0.001	0.000	0.000	0.000	0.000	0.000
R <sup>2</sup>	0.083	0.469	0.716	0.767	0.822	0.852
Regression Naive (n=2)						
c	0.023	0.019	0.015	0.040	0.045	0.032
Std. Dev.	0.044	0.057	0.050	0.054	0.058	0.054
p-value (c=0)	0.600	0.738	0.763	0.464	0.436	0.560
$\sigma_1$	0.299	0.846	1.254	1.170	1.194	1.387
Std. Dev.	0.076	0.085	0.076	0.073	0.079	0.074
p-value ( $\sigma_1=0$ )	0.000	0.000	0.000	0.000	0.000	0.000
$\sigma_2$	0.022	-0.265	-0.494	-0.368	-0.368	-0.532
Std. Dev.	0.076	0.082	0.075	0.073	0.076	0.071
p-value ( $\sigma_2=0$ )	0.773	0.002	0.000	0.000	0.000	0.000
R <sup>2</sup>	0.119	0.491	0.784	0.825	0.843	0.889
Regression Naive (n=3)						
c	0.003	0.008	0.016	0.019	0.011	0.007
Std. Dev.	0.039	0.053	0.050	0.051	0.051	0.050
p-value (c=0)	0.941	0.875	0.747	0.713	0.825	0.897
$\sigma_1$	0.501	0.941	1.298	1.223	1.187	1.371
Std. Dev.	0.077	0.081	0.089	0.083	0.077	0.082
p-value ( $\sigma_1=0$ )	0.000	0.000	0.000	0.000	0.000	0.000
$\sigma_2$	-0.114	-0.510	-0.608	-0.259	-0.078	-0.403
Std. Dev.	0.071	0.103	0.135	0.119	0.115	0.133
p-value ( $\sigma_2=0$ )	0.109	0.000	0.000	0.032	0.500	0.003
$\sigma_3$	0.219	0.323	0.090	-0.158	-0.264	-0.081
Std. Dev.	0.067	0.078	0.087	0.075	0.072	0.079
p-value ( $\sigma_3=0$ )	0.001	0.000	0.306	0.036	0.000	0.304
R <sup>2</sup>	0.307	0.577	0.781	0.841	0.879	0.906

Regression Naive (n=4)	1	2	3	4	5	6
<b>c</b>	0.015	0.039	0.042	0.038	0.028	0.022
<b>Std. Dev.</b>	0.036	0.038	0.044	0.046	0.046	0.043
<b>p-value (c=0)</b>	0.683	0.306	0.341	0.412	0.539	0.610
<b><math>\sigma_1</math></b>	0.679	1.182	1.283	1.274	1.261	1.514
<b>Std. Dev.</b>	0.083	0.063	0.078	0.081	0.081	0.076
<b>p-value (<math>\sigma_1=0</math>)</b>	0.000	0.000	0.000	0.000	0.000	0.000
<b><math>\sigma_2</math></b>	-0.329	-0.788	-0.606	-0.552	-0.339	-0.800
<b>Std. Dev.</b>	0.083	0.083	0.127	0.124	0.118	0.126
<b>p-value (<math>\sigma_2=0</math>)</b>	0.000	0.000	0.000	0.000	0.005	0.000
<b><math>\sigma_3</math></b>	0.310	0.480	0.014	0.261	0.076	0.403
<b>Std. Dev.</b>	0.067	0.080	0.127	0.110	0.103	0.117
<b>p-value (<math>\sigma_3=0</math>)</b>	0.000	0.000	0.915	0.019	0.461	0.001
<b><math>\sigma_4</math></b>	-0.126	-0.231	0.063	-0.246	-0.210	-0.283
<b>Std. Dev.</b>	0.065	0.059	0.077	0.069	0.068	0.067
<b>p-value (<math>\sigma_4=0</math>)</b>	0.055	0.000	0.411	0.001	0.003	0.000
<b>R<sup>2</sup></b>	0.398	0.765	0.814	0.862	0.900	0.932
<hr/>						
Regression Naive (n=5)						
<b>c</b>	0.031	0.030	0.030	0.022	0.022	0.020
<b>Std. Dev.</b>	0.034	0.037	0.041	0.042	0.046	0.043
<b>p-value (c=0)</b>	0.364	0.426	0.463	0.604	0.627	0.640
<b><math>\sigma_1</math></b>	0.537	1.341	1.385	1.454	1.331	1.532
<b>Std. Dev.</b>	0.082	0.088	0.082	0.082	0.090	0.092
<b>p-value (<math>\sigma_1=0</math>)</b>	0.000	0.000	0.000	0.000	0.000	0.000
<b><math>\sigma_2</math></b>	-0.077	-1.029	-0.705	-0.767	-0.439	-0.838
<b>Std. Dev.</b>	0.094	0.121	0.128	0.127	0.139	0.159
<b>p-value (<math>\sigma_2=0</math>)</b>	0.413	0.000	0.000	0.000	0.002	0.000
<b><math>\sigma_3</math></b>	0.046	0.677	-0.118	0.520	0.163	0.435
<b>Std. Dev.</b>	0.081	0.106	0.127	0.121	0.122	0.146
<b>p-value (<math>\sigma_3=0</math>)</b>	0.575	0.000	0.354	0.000	0.184	0.004
<b><math>\sigma_4</math></b>	-0.016	-0.387	0.489	-0.679	-0.316	-0.299
<b>Std. Dev.</b>	0.067	0.089	0.117	0.102	0.103	0.123
<b>p-value (<math>\sigma_4=0</math>)</b>	0.807	0.000	0.000	0.000	0.003	0.017
<b><math>\sigma_5</math></b>	-0.045	0.127	-0.331	0.325	0.082	0.008
<b>Std. Dev.</b>	0.061	0.061	0.071	0.065	0.071	0.072
<b>p-value (<math>\sigma_5=1</math>)</b>	0.459	0.040	0.000	0.000	0.250	0.911
<b>R<sup>2</sup></b>	0.332	0.726	0.828	0.882	0.899	0.930

**Table A.9**

This is the statistical analysis of the accumulated score for t-bill, having as reference  $A_t$ .

$\Sigma\pi$ (T-Bill)	1	2	3	4	5	6
<b>Total accumulated score</b>	3.760	61.770	72.630	72.710	96.300	88.440
<b>Mean</b>	3.503	39.042	45.540	41.610	54.976	50.344
<b>Standard deviation</b>	1.559	18.575	21.808	23.019	29.330	27.405
<b>max</b>	6.930	61.910	73.090	73.570	97.080	91.300
<b>min</b>	-0.140	-3.160	2.100	2.490	5.590	5.950
<b>Max Drawdown</b>	-6.610	-1.520	-3.250	-5.060	-9.510	-13.970
<b>constant standard deviation</b>	3.889	7.292	8.226	2.156	4.849	3.932
<b>p-value</b>	0.000	0.000	0.000	0.002	0.000	0.000
<b>coefficient standard deviation</b>	0.003	0.008	0.009	0.009	0.012	0.015
<b>p-value</b>	0.099	0.000	0.000	0.000	0.000	0.000
<b>R<sup>2</sup></b>	0.020	0.966	0.969	0.972	0.966	0.949

**Table A.10**

This is the statistical analysis of the accumulated score for t-bill, having as reference  $A_{t+1}$ .

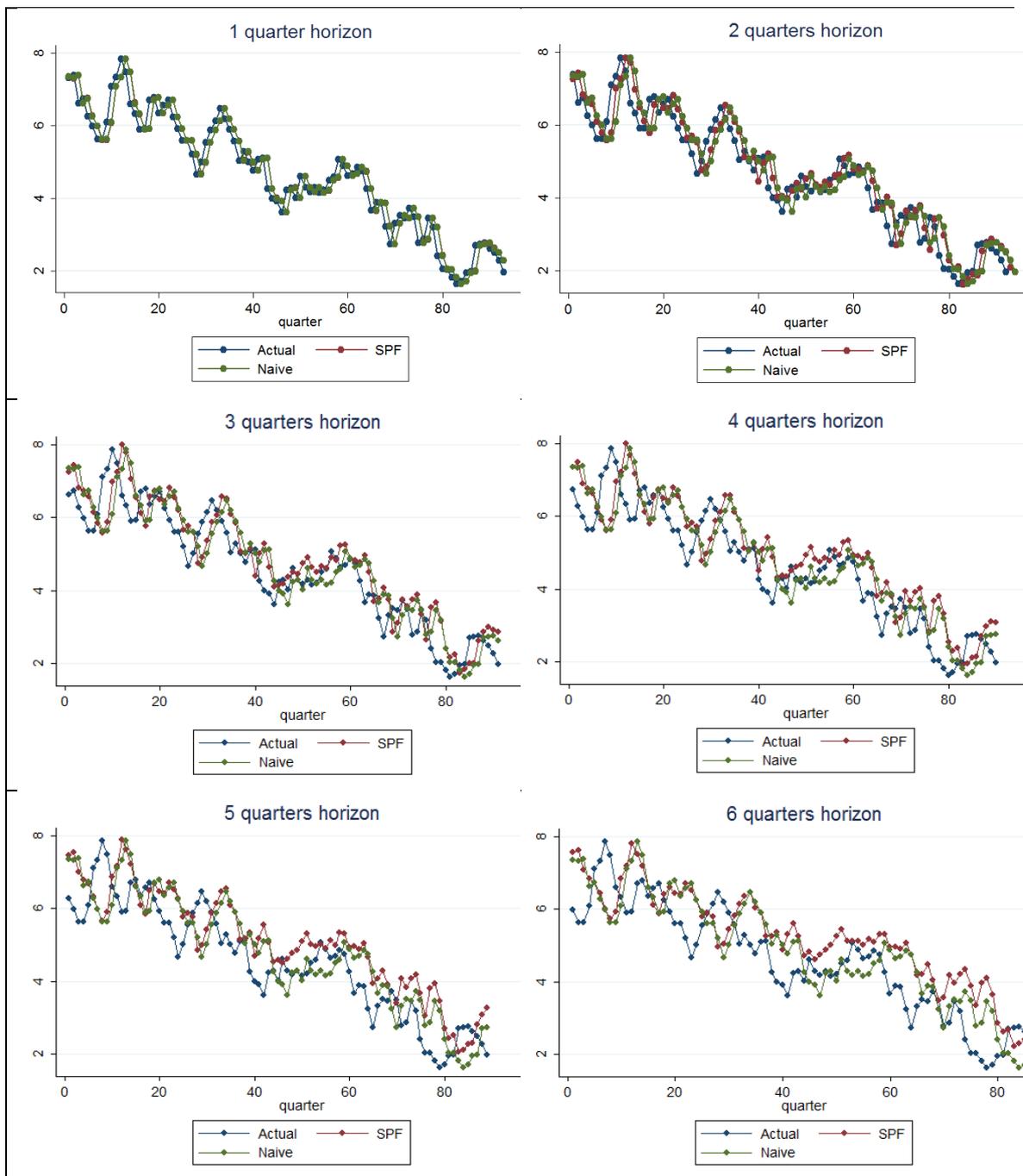
$\Sigma\pi'$ (T-Bill)	1	2	3	4	5	6
<b>Total accumulated score</b>		16.970	30.110	33.330	59.380	55.000
<b>Mean</b>		9.190	17.181	15.474	30.595	50.344
<b>Standard deviation</b>		6.781	10.715	13.093	20.060	27.405
<b>max</b>		18.200	31.490	34.970	60.940	91.300
<b>min</b>		-4.750	-1.920	-4.780	-1.900	5.950
<b>Max Drawdown</b>		-2.310	-7.970	-10.170	-11.470	-16.690
<b>constant standard deviation</b>		-2.295	-0.903	-6.323	-3.148	-3.913
<b>p-value</b>		0.000	0.047	0.000	0.001	0.000
<b>coefficient standard deviation</b>		0.170	0.270	0.328	0.511	0.489
<b>p-value</b>		0.003	0.006	0.009	0.012	0.014
<b>p-value</b>		0.000	0.000	0.000	0.000	0.000
<b>R<sup>2</sup></b>		0.949	0.943	0.917	0.936	0.901

## Appendix 2

In annex I we find all the tables and graphs related with the analysis of the error forecasts of 10 years Treasury Bond.

**Table A.11**

These are graphs of the comparison of the actual value ( $A_{t+f}$ ), the mean of SPF ( $P_{t+f}$ ) and the naïve forecast ( $A_t$ ) for the 10 year Treasury Bond.



**Table A.12**

It is a statistical description of the forecast error for the survey professional of forecasters and the naïve forecast for the long term interest rate.

<b>SPF</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>
<b>Observations</b>	93	92	91	90	89	88
<b>Mean forecast error</b>	-0.058	-0.133	-0.272	-0.406	-0.537	-0.656
<b>Std. Dev. Forecast error</b>	0.382	0.491	0.663	0.762	0.842	0.871
<b>Min forecast error</b>	-0.880	-1.116	-1.610	-1.775	-2.116	-2.463
<b>Max forecast error</b>	1.019	1.308	1.514	1.951	2.172	1.849
<b>Naïve</b>						
<b>Mean actual-X error</b>	-0.058	-0.113	-0.168	-0.214	-0.261	-0.304
<b>Std. Dev. Actual-X error</b>	0.380	0.590	0.717	0.827	0.896	0.915
<b>Min actual-X error</b>	-0.870	-1.240	-1.570	-1.940	-1.930	-2.040
<b>Max actual-X error</b>	1.000	1.470	1.750	2.220	2.220	1.850

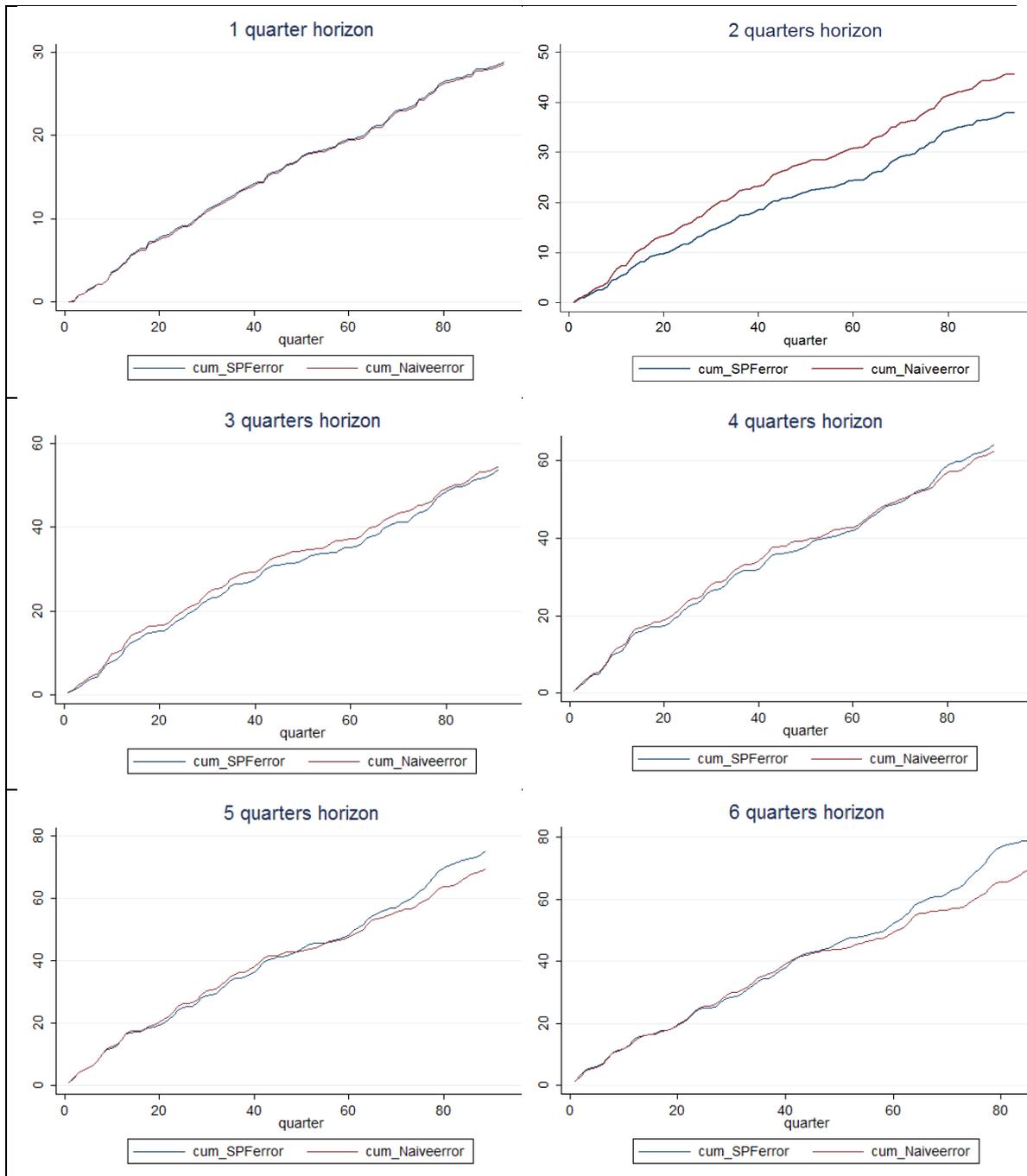
**Table A.13**

It is a statistical description of the forecast errors in absolute values for the survey professional of forecasters and the naïve forecast for the long term interest rate.

<b>SPF</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>
<b>Observations</b>	93	92	91	90	89	88
<b>Mean ABS-forecast error</b>	0.308	0.412	0.590	0.716	0.845	0.919
<b>Std. Dev. ABS Forecast error</b>	0.229	0.294	0.402	0.480	0.527	0.584
<b>Min ABS forecast error</b>	1.000	1.308	1.610	1.951	2.172	2.463
<b>Max ABS forecast error</b>	0.000	0.032	0.027	0.015	0.023	0.012
<b>Naïve</b>						
<b>Mean ABS actual-X error</b>	0.310	0.495	0.599	0.695	0.778	0.805
<b>Std. Dev. ABS Actual-X error</b>	0.229	0.336	0.423	0.492	0.510	0.524
<b>Min ABS actual-X error</b>	1.019	1.470	1.750	2.220	2.220	2.040
<b>Max ABS actual-X error</b>	0.0012	0	0.01	0.01	0	0.03

**Table A.14**

It is the graphs of accumulated errors in absolute value for SPF and naïve forecasts for the long term interest rate.



**Table A.15**

It is the estimation of model (1) and (3) for forecasts on 10 years Treasury Bond.

Regression (1)	1	2	3	4	5	6
$\alpha$	0.077	0.064	0.065	-0.029	-0.152	-0.418
Std. Dev.	0.124	0.164	0.233	0.285	0.338	0.378
p-value ( $\alpha=0$ )	0.538	0.699	0.780	0.920	0.654	0.271
$\beta$	0.971	0.959	0.930	0.924	0.924	0.954
Std. Dev.	0.025	0.033	0.046	0.055	0.064	0.071
p-value ( $\beta=0$ )	0.000	0.000	0.000	0.000	0.000	0.000
t-value ( $\beta=1$ )	-1.141	-1.256	-1.516	-1.379	-1.183	-0.651
p-value ( $\beta=1$ )	0.257	0.212	0.133	0.171	0.240	0.517
R <sup>2</sup>	0.943	0.904	0.822	0.762	0.705	0.679
Regression B	1	2	3	4	5	6
$\alpha'$	-0.058	-0.133	-0.272	-0.406	-0.537	-0.656
Std. Dev.	0.040	0.051	0.069	0.080	0.089	0.093
p-value ( $\alpha'=0$ )	0.149	0.011	0.000	0.000	0.000	0.000
R <sup>2</sup>	0.000	0.000	0.000	0.000	0.000	0.000

**Table A.16**

It is the estimation of model (2) and (4) for forecasts on 10 years Treasury Bond.

Regression A2	1	2	3	4	5	6
$\delta$	0.076	0.230	0.371	0.496	0.586	0.574
Std. Dev.	0.124	0.193	0.235	0.273	0.297	0.308
p-value ( $\delta=0$ )	0.543	0.237	0.119	0.073	0.052	0.066
$\lambda$	0.972	0.927	0.887	0.851	0.824	0.818
Std. Dev.	0.025	0.039	0.047	0.054	0.059	0.061
p-value ( $\lambda=0$ )	0.000	0.000	0.000	0.000	0.000	0.000
t-value ( $\lambda=1$ )	-1.137	-1.873	-2.410	-2.736	-2.991	-2.995
p-value ( $\lambda=1$ )	0.258	0.064	0.018	0.007	0.004	0.004
R <sup>2</sup>	0.943	0.864	0.800	0.736	0.692	0.678
Regression B2	1	2	3	4	5	6
$\delta'$	-0.058	-0.113	-0.168	-0.214	-0.261	-0.304
Std. Dev.	0.039	0.061	0.075	0.087	0.095	0.098
p-value ( $\delta'=0$ )	0.146	0.069	0.028	0.016	0.007	0.002
R <sup>2</sup>	0.000	0.000	0.000	0.000	0.000	0.000

**Table A.17**

In these tables we can find the results of the estimation of model (5) for fulfilling the table 6.2.2.

Regression SPF (n=1)	1	2	3	4	5	6
c	-0.001	-0.004	0.000	-0.004	0.001	0.005
Std. Dev.	0.039	0.050	0.057	0.055	0.060	0.059
p-value (c=0)	0.987	0.934	0.998	0.946	0.986	0.926
$\varphi_1$	0.104	0.302	0.588	0.743	0.746	0.775
Std. Dev.	1.940	0.101	0.087	0.072	0.072	0.068
p-value ( $\varphi_1=0$ )	-0.004	0.004	0.000	0.000	0.000	0.000
R <sup>2</sup>	0.040	0.091	0.344	0.547	0.556	0.606
Regression SPF (n=2)	1	2	3	4	5	6
c	-0.001	0.006	0.005	0.007	0.007	0.010
Std. Dev.	0.039	0.047	0.051	0.043	0.055	0.053
p-value (c=0)	0.970	0.897	0.928	0.872	0.899	0.847
$\varphi_1$	0.237	0.397	0.874	1.198	1.074	1.132
Std. Dev.	0.105	0.101	0.094	0.084	0.098	0.097
p-value ( $\varphi_1=0$ )	0.026	0.000	0.000	0.000	0.000	0.000
$\varphi_2$	-0.182	-0.292	-0.493	-0.620	-0.446	-0.466
Std. Dev.	0.105	0.101	0.094	0.085	0.097	0.096
p-value ( $\varphi_2=0$ )	0.087	0.005	0.000	0.000	0.000	0.000
R <sup>2</sup>	0.072	0.176	0.501	0.724	0.642	0.689
Regression SPF (n=3)	1	2	3	4	5	6
c	0.006	0.001	0.005	0.005	0.008	0.007
Std. Dev.	0.039	0.048	0.051	0.043	0.055	0.053
p-value (c=0)	0.872	0.990	0.927	0.912	0.887	0.901
$\varphi_1$	0.260	0.448	0.929	1.075	0.995	1.141
Std. Dev.	0.105	0.108	0.109	0.107	0.109	0.111
p-value ( $\varphi_1=0$ )	0.015	0.000	0.000	0.000	0.000	0.000
$\varphi_2$	-0.202	-0.346	-0.591	-0.372	-0.256	-0.472
Std. Dev.	0.106	0.110	0.134	0.153	0.152	0.159
p-value ( $\varphi_2=0$ )	0.061	0.002	0.000	0.017	0.096	0.004
$\varphi_3$	0.082	0.116	0.113	-0.211	-0.176	0.004
Std. Dev.	0.105	0.106	0.109	0.106	0.109	0.110
p-value ( $\varphi_3=0$ )	0.439	0.277	0.303	0.051	0.108	0.974
R <sup>2</sup>	0.085	0.195	0.506	0.737	0.649	0.687

Regression SPF (n=4)	1	2	3	4	5	6
c	0.003	0.011	0.012	0.011	0.008	0.001
Std. Dev.	0.038	0.045	0.048	0.043	0.049	0.049
p-value (c=0)	0.937	0.805	0.799	0.798	0.866	0.980
$\varphi_1$	0.306	0.501	0.971	1.086	0.914	1.140
Std. Dev.	0.105	0.102	0.102	0.110	0.099	0.101
p-value ( $\varphi_1=0$ )	0.004	0.000	0.000	0.000	0.000	0.000
$\varphi_2$	-0.265	-0.495	-0.824	-0.405	-0.379	-0.666
Std. Dev.	0.106	0.111	0.138	0.160	0.138	0.153
p-value ( $\varphi_2=0$ )	0.014	0.000	0.000	0.013	0.007	0.000
$\varphi_3$	0.152	0.279	0.472	-0.187	0.295	0.495
Std. Dev.	0.105	0.109	0.138	0.158	0.138	0.153
p-value ( $\varphi_3=0$ )	0.154	0.012	0.001	0.242	0.036	0.002
$\varphi_4$	-0.258	-0.341	-0.391	-0.007	-0.472	-0.433
Std. Dev.	0.102	0.100	0.102	0.109	0.099	0.101
p-value ( $\varphi_4=0$ )	0.014	0.001	0.000	0.950	0.000	0.000
R <sup>2</sup>	0.159	0.305	0.583	0.740	0.725	0.747
Regression SPF (n=5)	1	2	3	4	5	6
$\alpha''$	0.012	0.013	0.018	0.013	0.006	-0.010
Std. Dev.	0.037	0.045	0.048	0.044	0.049	0.048
p-value $\alpha''$	0.745	0.773	0.704	0.762	0.904	0.835
$\rho_1$	0.288	0.448	0.922	1.080	1.025	1.092
Std. Dev.	0.106	0.110	0.111	0.112	0.110	0.111
p-value $\rho_1$	0.008	0.000	0.000	0.000	0.000	0.000
$\rho_2$	-0.291	-0.453	-0.769	-0.418	-0.449	-0.623
Std. Dev.	0.107	0.116	0.148	0.164	0.140	0.161
p-value $\rho_2$	0.008	0.000	0.000	0.013	0.002	0.000
$\rho_3$	0.134	0.207	0.381	-0.242	0.385	0.468
Std. Dev.	0.107	0.124	0.166	0.167	0.142	0.169
p-value $\rho_3$	0.213	0.098	0.024	0.151	0.008	0.007
$\rho_4$	-0.227	-0.271	-0.294	0.121	-0.684	-0.343
Std. Dev.	0.104	0.113	0.148	0.161	0.140	0.161
p-value $\rho_4$	0.032	0.019	0.050	0.453	0.000	0.036
$\rho_5$	-0.159	-0.147	-0.102	-0.117	0.232	-0.089
Std. Dev.	0.103	0.107	0.111	0.111	0.110	0.111
p-value $\rho_5$	0.129	0.172	0.358	0.294	0.039	0.423
R <sup>2</sup>	0.210	0.314	0.587	0.743	0.740	0.757

**Table A.18**

In these tables we can find the results of the estimation of model (6) for fulfilling the table 6.2.2.

Regression Naive (n=1)	1	2	3	4	5	6
c	-0.001	-0.004	0.002	0.000	0.005	0.008
Std. Dev.	0.039	0.053	0.053	0.059	0.062	0.059
p-value (c=0)	0.985	0.941	0.964	0.994	0.936	0.891
$\sigma_1$	0.194	0.524	0.722	0.747	0.763	0.795
Std. Dev.	0.104	0.091	0.074	0.072	0.069	0.065
p-value ( $\sigma_1=0$ )	0.065	0.000	0.000	0.000	0.000	0.000
R <sup>2</sup>	0.037	0.273	0.522	0.556	0.588	0.640
Regression Naive (n=2)	1	2	3	4	5	6
c	-0.001	0.006	0.003	0.006	0.008	0.006
Std. Dev.	0.039	0.049	0.043	0.053	0.056	0.050
p-value (c=0)	0.972	0.910	0.950	0.906	0.884	0.903
$\sigma_1$	0.228	0.744	1.151	1.097	1.104	1.225
Std. Dev.	0.105	0.097	0.087	0.095	0.097	0.094
p-value ( $\sigma_1=0$ )	0.032	0.000	0.000	0.000	0.000	0.000
$\sigma_2$	-0.182	-0.417	-0.594	-0.474	-0.451	-0.541
Std. Dev.	0.105	0.097	0.087	0.095	0.097	0.093
p-value ( $\sigma_2=0$ )	0.086	0.000	0.000	0.000	0.000	0.000
R <sup>2</sup>	0.069	0.406	0.690	0.656	0.667	0.740
Regression Naive (n=3)	1	2	3	4	5	6
c	0.006	0.004	0.012	0.012	0.016	0.011
Std. Dev.	0.039	0.049	0.043	0.052	0.056	0.051
p-value (c=0)	0.872	0.930	0.771	0.811	0.777	0.837
$\sigma_1$	0.252	0.826	1.097	0.983	1.044	1.233
Std. Dev.	0.105	0.107	0.106	0.107	0.109	0.112
p-value ( $\sigma_1=0$ )	0.019	0.000	0.000	0.000	0.000	0.000
$\sigma_2$	-0.204	-0.562	-0.501	-0.229	-0.330	-0.576
Std. Dev.	0.106	0.124	0.149	0.150	0.155	0.167
p-value ( $\sigma_2=0$ )	0.059	0.000	0.001	0.130	0.036	0.001
$\sigma_3$	0.091	0.196	-0.090	-0.219	-0.109	0.029
Std. Dev.	0.105	0.106	0.106	0.106	0.109	0.111
p-value ( $\sigma_3=0$ )	0.388	0.067	0.395	0.042	0.321	0.792
R <sup>2</sup>	0.083	0.424	0.701	0.669	0.667	0.734

Regression Naive (n=4)	1	2	3	4	5	6
c	0.003	0.010	0.008	0.011	0.008	-0.006
Std. Dev.	0.038	0.043	0.043	0.047	0.050	0.046
p-value (c=0)	0.934	0.823	0.849	0.817	0.870	0.901
$\sigma_1$	0.299	0.924	1.121	0.887	1.004	1.272
Std. Dev.	0.105	0.095	0.110	0.099	0.098	0.100
p-value ( $\sigma_1=0$ )	0.005	0.000	0.000	0.000	0.000	0.000
$\sigma_2$	-0.266	-0.850	-0.505	-0.340	-0.491	-0.842
Std. Dev.	0.105	0.122	0.161	0.137	0.141	0.159
p-value ( $\sigma_2=0$ )	0.014	0.000	0.002	0.015	0.001	0.000
$\sigma_3$	0.158	0.607	-0.116	0.241	0.398	0.593
Std. Dev.	0.105	0.122	0.160	0.137	0.142	0.159
p-value ( $\sigma_3=0$ )	0.136	0.000	0.469	0.083	0.006	0.000
$\sigma_4$	-0.257	-0.494	0.038	-0.461	-0.474	-0.445
Std. Dev.	0.103	0.095	0.107	0.099	0.097	0.100
p-value ( $\sigma_4=0$ )	0.014	0.000	0.722	0.000	0.000	0.000
R <sup>2</sup>	0.156	0.566	0.703	0.736	0.743	0.795
Regression Naive (n=5)	1	2	3	4	5	6
c	0.012	0.014	0.018	0.017	0.013	-0.008
Std. Dev.	0.037	0.043	0.039	0.043	0.051	0.046
p-value (c=0)	0.748	0.751	0.648	0.690	0.803	0.855
$\sigma_1$	0.280	0.978	1.165	1.085	1.002	1.210
Std. Dev.	0.107	0.110	0.099	0.100	0.113	0.114
p-value ( $\sigma_1=0$ )	0.010	0.000	0.000	0.000	0.000	0.000
$\sigma_2$	-0.288	-0.919	-0.645	-0.458	-0.503	-0.757
Std. Dev.	0.107	0.139	0.148	0.126	0.151	0.177
p-value ( $\sigma_2=0$ )	0.009	0.000	0.000	0.000	0.001	0.000
$\sigma_3$	0.138	0.692	-0.215	0.390	0.398	0.480
Std. Dev.	0.107	0.153	0.153	0.128	0.154	0.187
p-value ( $\sigma_3=0$ )	0.199	0.000	0.163	0.003	0.012	0.012
$\sigma_4$	-0.226	-0.592	0.414	-0.844	-0.460	-0.280
Std. Dev.	0.104	0.138	0.145	0.127	0.150	0.174
p-value ( $\sigma_4=0$ )	0.033	0.000	0.006	0.000	0.003	0.112
$\sigma_5$	-0.157	0.116	-0.383	0.427	-0.018	-0.130
Std. Dev.	0.104	0.109	0.098	0.100	0.113	0.112
p-value ( $\sigma_5=1$ )	0.134	0.288	0.000	0.000	0.874	0.250
R <sup>2</sup>	0.205	0.573	0.764	0.787	0.744	0.797

**Table 19**

This is the statistical analysis of the accumulated score for T-bond, having as reference  $A_t$ .

<b>Cum results</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>
<b>Total accumulated score</b>	-6.380	20.170	11.630	3.370	1.680	-2.380
<b>Mean deviation</b>	-5.444	14.758	10.095	4.786	3.741	3.486
<b>max</b>	-0.040	21.680	15.130	9.210	7.580	9.700
<b>min</b>	-7.660	-0.720	0.160	-1.570	-2.410	-6.100
<b>Max Drawdown constant</b>	-7.620	-1.910	-4.950	-8.080	-8.920	-15.800
<b>standard deviation</b>	-3.523	3.822	4.497	2.676	2.923	5.248
<b>p-value</b>	0.246	0.544	0.512	0.486	0.524	0.878
<b>coefficient</b>	0	0	0	0	0	0
<b>standard deviation</b>	-0.041	0.235	0.122	0.046	0.018	-0.040
<b>p-value</b>	0.005	0.010	0.010	0.009	0.010	0.017
<b>R^2</b>	0	0	0	0	0.076	0.023
	0.4707	0.8564	0.6409	0.2209	0.0357	0.0585

**Table 20**

This is the statistical analysis of the accumulated score for T-bond, having as reference  $A_{t+1}$ .

<b>Cum results2</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>
<b>Total accumulated score</b>		-2.470	-7.490	-11.210	-8.800	-11.280
<b>Mean deviation</b>		0.945	-1.966	-5.844	-4.816	-4.509
<b>max</b>		1.642	2.862	3.347	2.924	4.291
<b>min</b>		3.630	1.880	0.570	0.600	2.360
<b>Max Drawdown constant</b>		-2.470	-8.610	-13.430	-11.840	-15.070
<b>standard deviation</b>		-6.100	-10.490	-14.000	-12.440	-17.430
<b>p-value</b>		1.737	1.867	-0.664	-0.377	1.560
<b>coefficient</b>		0.334	0.389	0.328	0.308	0.542
<b>standard</b>		0	0	0.046	0.225	0.005
		-0.017	-0.083	-0.114	-0.099	-0.136
		0.006	0.007	0.006	0.006	0.011

<b>deviation</b>					
<b>p-value</b>	0.008	0	0	0	0
<b>R<sup>2</sup></b>	0.0766	0.5912	0.7895	0.7596	0.6593