How Do Inflation Forecasts Affect the Estimation of DSGE Models?

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Abstract

In this paper we analyse the consequences of considering inflation forecasts from the Survey of Professional Forecasters (SPF) as observables in the estimation of the Smets and Wouters (2007) DSGE model. We analyse how the model performs by studying a few alternative specifications of the monetary policy rule. Moreover, by allowing for persistence in the discrepancy between the rational expectation of inflation derived from the model and the corresponding inflation forecast from the SPF, we are able to assess the informational content of the SPF inflation forecasts. We conclude that SPF do not support the rationality assumption.

1 Introduction

Expectations play a crucial role in economics. Agents' beliefs about what is going to happen in the future affect all economic variables from real variables (such as output, consumption, investment, and wages) to nominal variables like inflation and nominal interest rate.

In modern macroeconomics, expectations - specifically inflation expectations - drive price adjustment mechanism. More precisely, the Calvo's(1983) sticky-price scheme results in the forward-looking New-Keynesian Phillips curve. The so-called Calvo lottery leads to a situation where firms, which are able to adjust prices, establish these prices maximizing the expected present value of current and future profits. Therefore, agents' beliefs on future prices are a cornerstone of modern macroeconomics models.Similarly, household are forward-looking agents whose intertemporal decisions are largely driven by their beliefs.

One of the most widely extended assumption in economics is that expectations are rational, which means agents are able to roughly anticipate the future values of economic variables beyond some small and unpredictable forecasting errors. Nevertheless, looking at the difference between the observed time series on inflation and the survey expectations data (for instance the SPF conducted by The Federal Reserve Bank of Philadelphia or the Survey of Consumers reported by the University of Michigan), we find some consistent discrepancies between actual and forecasted time series. Therefore, one may think that data on inflation expectations may contain useful information to improve model parameter identification.

The literature studying the statistical properties of survey data on expectations is rather controversial. Several authors support the non-rationality of the surveys' forecasts. Batchelor (2007) documents the presence of systematic bias in the real GDP and inflation forecasts of private sector forecasters in the G7 countries. Souleles (2004) provides evidence on the existence of a non-constant bias and inefficiency in inflation expectations of household data. Bialowolski (2015) assesses how the information set which households consider in their inflation forecasting leads to a bias forecast on the household inflation expectations data. Bryan and Palmqvist (2005) find that surveys of household inflation expectations for the US and Sweden do not support the assumption of rationality. Bakhshi and Yates (1998) find that the survey data of inflation expectations of UK employees systematically overestimate inflation. Bentancor and Pincheira (2008) find remarkable autocorrelation and bias in the SPF of the Central Bank of Chile over the period 2000-2008. Finally, a more related to the aim of this paper, Diebold et. al (1997) provide evidence on the high values of autocorrelation present in the SPF for US inflation expectations. In a similar vein, Mehra (2003) concludes that forecasters from the SPF are biased. He also shows that survey data Granger-cause inflation. Therefore, inflation survey data contains additional information, which might be important in the estimation of the models.

In this line, this paper investigates whether including observed data of inflation expectations in the estimation of macroeconomics models helps. The consideration of survey data on inflation expectations allows us to take advantage of the informational content of these data. Moreover, looking at different indicators we can study the plausibility of assuming rationality in the formation of the SPF.

We consider the DSGE model of Smets and Wouters (2007). This model is widely used not only in the economic literature, but also large-scale versions of this model are being used increasingly by central banks for policy analysis. Following Del Negro and Eusepi (2011), we include inflation expectations data in the set of observable variables. Thus, beyond a measurement error, survey data on inflation is identified with inflation expectations from the model. We consider inflation expectations for different horizons and conclude that the best-fit specification considers survey data for the current (now-cast), the one-quarter-ahead and the four-quarter-ahead horizons. Moreover, we consider different specifications for the monetary policy reaction function, where the nominal interest rate responds either to current inflation or inflation expectations defined for alternative forecast horizons, concluding that the best fit is obtained for the policy rule considering the one-quarter-ahead inflation expectations.

Furthermore, we extend our analysis by implementing a new specification for the price and wage mark-up shock processes suggested by Slobodyan and Wouters (2017). This specification captures more accurately the less volatile behaviour of inflation expectations. In line with Slobodyan and Wouters (2017), we find that this specification leads to a large improvement of the model fit.

We carry out a comparison exercise among the different specifications of the model throughout a Bayesian estimation approach. First of all we analyse the robustness of parmeters estimation, by estimating four alternative specifications: (1) including inflation forecast from SPF as observables, (2) implementing the Slobodyan and Wouters (2017) specification for the price and wage mark-up shock processes, (3) incorporating inflation forecasts at alternative forecast horizons in the policy rule, (4) allowing the discrepancy between the inflation forecast reported in the

SPF and the corresponding inflation expectation defined in the model to follow a persistent process. Regarding this last specification where we consider a persistent discrepancy between model's inflation expectations and inflation forecasts from the SPF, we show that the fit of the model improves by relaxing the assumption that SPF's inflation forecasts are consistent with the rational expectations assumed by the model. This exercise can be considered as an indirect test for rationality of inflation expectations based on a DSGE specification and inflation forecast data.

In this paper, we focus on the SPF which was conducted for the first time in 1968 by the American Statistical Association and the National Bureau of Economic Research. Since 1990 the responsibility of carrying out this survey has been undertaken by The Federal Reserve Bank of Philadelphia. We choose this survey data because of its early availability and its reliability. Some authors (e.g. , Keane and Runkle, 1990), claim that professional forecasters have more incentives to make an accurate forecast than participants in other surveys. In addition, professionals report quantitative data while household surveys report only qualitative expectations.

In addition to the standard goodness-of-fit measure based on the log-marginal likelihood, comparison of the alternative specifications of the medium-scale New-Keynesian model is carried out by looking at the posterior second moments generated through the Bayesian estimation of the alternative specifications.In this way, we can uncover which specifications do a good a job in reproducing the business cycle features exhibited by actual US data.

The structure of the paper is as follows. Section 2 provides a brief description of the Smet and Wouters (2007) model and the specification proposed by Slobodyan and Wouters (2017). Moreover, this section introduces the different specifications we take into consideration for the monetary policy reaction function. Section 3 presents the estimation results and discuss all the differences found across the specifications we have presented in the previous section. Finally, in section 4 we conclude.

2 The model

We consider the medium-scale New Keynesian model proposed by Smets and Wouters (2007). The main feature of the model is the existence of nominal and real rigidities. For instance the price and wage stickiness are both determined by a Calvo lottery. Another noteworthy feature of the model is that the fluctuations of the nominal and real variables are driven by several shocks that affect firms and households decisions.

The supply side of the economy is composed by two type of firms, the final and the intermediate sector. Firms in the final good sector produce good Y_t by a blend of intermediate goods and they sell it in a perfectly competitive market.

The intermediate sector is populated by a continuum of firms which hire labour and rent capital at certain real wages and rental rates of capital . Intermediate goods producers sell their output in a non-competitive market. Market power is introduced in order to accommodate price rigidity.

The labour market is also differentiated in two tiers: labour unions and labour packers. Households supply labour to unions which differentiate it and establish wages according to a Calvo lottery mechanism, as it is done in the intermediate good sector. This wage mechanism introduces wage stickiness. Labour unions sell heterogeneous labour to labour packers who package and offer it to intermediate goods producers. Both wage and price mark-up profits are distributed among households as dividends.

Households decide on their consumption, labour supply, investment, capital utilization and bonds purchase. The smoothness behaviour of consumption is captured by the model throughout an external habit formation component.

Next, we present the log-linearised DSGE model. All the variables are expressed in log-deviation around their balanced-growth-path steady-state values. A full description of the model can be found in the appendix of Smets and Wouters (2007).

Supply side

The log-linearised aggregate production function faced by firms is given by:

$$y_t = \phi_p[\alpha k_t^s + (1-\alpha)]l_t + \epsilon_t^a \tag{1}$$

Output is produced by the combination of two inputs, labour l_t and capital k_t^s. Parameter α represents the capital income share and ϕ_p is one plus the proportion of fixed cost in production. Total factor productivity ϵ_t^a follows a first-order autoregressive process ($\epsilon_t^a = \rho_a \epsilon_{t-1}^a + \eta_t^a$). Capital used in production function k_t^s, is a proportion of the capital stock in the previous period determined by the degree of capital utilization z_t:

$$k_t^s = k_{t-1} + z_t \tag{2}$$

The degree of capital utilization is determined in the cost minimization problem faced by the households. Therefore, the degree of capital utilization is a positive function of the returns of capital:

$$z_t = \frac{(1-\Psi)}{\Psi} r_t^k \tag{3}$$

where Ψ represents the elasticity of the capital utilization adjustment cost function and it takes values from 0 (marginal cost of changing the capital is constant) to 1 (to change in the utilization of capital is too costly, therefore it remains constant).

The law of motion of the capital depends on the net investment and on the relative efficiency of that investment represented by investment-specific technology disturbance ϵ_t^i,which follows a first order autoregressive process ($\epsilon_t^i = \rho_i \epsilon_{t-1}^a + \eta_t^i$).

$$k_t = \frac{1-\delta}{\gamma}\, k_{t-1} + \left[1 - \frac{1-\delta}{\gamma}\right] i_t + \left[1 - (\frac{1-\delta}{\gamma})(1+\beta\gamma^{1-\sigma_c})\gamma^2\varphi\right] \epsilon_t^i \tag{4}$$

The model considers market power in labour and good markets. As a consequence, there is a price mark-up (μ_t^p, which is the difference between the marginal product of labour mpl_t and real wage w_t) and a wage mark-up (μ_t^w, which is the difference between the real wage and the marginal rate of substitution between labour and consumption mrs_t):

$$\mu_t^p = mpl_t - w_t = \left[\alpha(k_t^s - l_t) + \epsilon_t^a\right] - w_t \tag{5}$$

$$\mu_t^w = w_t - mrs_t = w_t - \left[\sigma_l l_t + \frac{1}{1-\lambda/\gamma}(c_t - \lambda/\gamma c_{t-1})\right] \tag{6}$$

where σ_l represents the elasticity of labour supply with respect to real wage and λ represents the degree of external habits indexation in the consumption decision.

Due to the consideration of lagged inflation indexation and the price rigidity introduced by the Calvo lottey scheme, inflation π_t depends positively on past and future inflation and negatively on the price mark-up:

$$\pi_t = \pi_1 \pi_{t-1} + \pi_2 \pi_{t+1} - \pi_3 \mu_t^p + \epsilon_t^p \tag{7}$$

where $\pi_3 = \left[\xi_p((\phi_p - 1)\epsilon_p + 1)/(1 + \beta\gamma^{1-\sigma_c}\iota_p)(1 - \beta\gamma^{1-\sigma_c}\xi_p)(1 - \xi_p)^{-1} \right]$, $\pi_1 = \iota_p/1 + \beta\gamma^{1-\sigma_c}\iota_p$ and $\pi_2 = \beta\gamma^{1-\sigma_c}/1 + \beta\gamma^{1-\sigma_c}\iota_p$. Parameter ι_p represents the degree of indexation to past inflation, while parameter ξ_p represents the degree of price stickiness (i.e. Calvo's probability). Moreover, Smets and Wouters (2007) assume that the price mark-up process follows a first-order autoregressive moving average process ($\epsilon_t^p = \rho_p \epsilon_{t-1}^p + \mu_p \eta_{t-1}^p + \eta_t^p$). The MA component is included to capture the high frequency volatility of inflation. However, following Slobodyan and Wouters (2017), we will consider an alternative specification at the end of this section.

From the optimization problem of the firms we determine the rental rate of capital as the difference between real wages and the capital-labour ratio:

$$r_t^k = w_t - (k_t - l_t) \tag{8}$$

Analogically to the price determination mechanism, wages are determined by expected and future wages, past, current and expected inflation and wage mark-ups:

$$w_t = w_1 w_{t-1} + (1 - w_1)(E_t w_{t+1} + E_t \pi_{t+1}) - w_2 \pi_t + w_3 \pi_{t-1} - w_4 \mu_t^w + \epsilon_t^w \tag{9}$$

where $w_1 = 1/(1 + \beta\gamma^{1-\sigma_c})$, $w_2 = (1 + \beta\gamma^{1-\sigma_c}\iota_w)/(1 + \beta\gamma^{1-\sigma_c})$, $w_3 = \iota_w/(1 + \beta\gamma^{1-\sigma_c})$ and $w_4 = 1/(1 + \beta\gamma^{1-\sigma_c})[(1 - \beta\gamma^{1-\sigma_c}\xi_w)(1 - \xi_w)/(\xi_w((\phi_w - 1)\epsilon_w + 1))]$. This equilibrium equation considers a wage mark-up disturbance which follows an ARMA(1,1) process to capture the persistent and the transitory shocks which affect the variations of wages ($\epsilon_t^w = \rho_w \epsilon_{t-1}^w + \mu_w \eta_{t-1}^w + \eta_t^w$). As with price mark-up process we will consider an alternative specification below. Parameters ι_w and ξ_w represent the degree of indexation of past wages and the degree of wage stickiness (i.e. wage Calvo probability), respectively.

Demand side

The optimization problem of the consumers results in the following Euler equation which determines the optimum consumption path over time:

$$c_t = c_1 c_{t-1} + (1 - c_1)E_t c_{t+1} + c_2(l_t - E_t l_{t+1}) - c_3(r_t - E_t \pi_{t+1} + \epsilon_t^b) \tag{10}$$

where $c_1 = (\lambda/\gamma)/(1+\lambda/\gamma)$, $c_2 = [(\sigma_c - 1)(W^h L/C)]/[\sigma_c(1+\lambda/\gamma)$ and $c_3 = (1-\lambda/\gamma)/[(1+\lambda/\gamma)\sigma_c]]$. Current consumption depends on past and future consumption, the evolution of labour hours, the real interest rate and on the gap between the interest rate established by the central bank and the return on assets represented by disturbance ϵ_t^b . A positive shock on this gap increases incentives to hold assets, decreasing current consumption. This disturbance is assumed to follow a first-order autoregressive process ($\epsilon_t^b = \rho_b \epsilon_{t-1}^b + \eta_t^b$).

The investment Euler equation depends positively on past and expected investment, the real value of existing capital stock and an investment-specific technology disturbance:

$$i_t = i_1 i_{t-1} + (1 - i_1)E_t i_{t-1} + i_2 q_t + \epsilon_t^i \tag{11}$$

where $i_1 = (1 + \beta\gamma^{1-\sigma_c})$ and $i_2 = [1/(1+\beta\gamma^{1-\sigma_c})\gamma^2\varphi]$. Parameter β is the discount factor and φ is the steady-state elasticity of the capital adjustment cost and presents a positive relationship with the sensitivity of investment to real value of existing capital stock.

The current value of capital depends positively on its own expected value and the expected value of the rental rate of capital and negatively on the expected real interest rate and the risk premium disturbance:

$$q_t = q_1 E_t q_{t+1} + (1 - q_1)E_t r_{t+1}^k - (r_t - E_t \pi_{t+1} + \epsilon_t^b) \tag{12}$$

where $q_1 = \beta\gamma^{\sigma_c}(1-\delta)$. A positive shock on the risk premium disturbance increases the cost of capital and consequently decrease the incentives to invest in capital.

The resource constraint

The resource constraint describes the alternative uses of output y_t: consumption (c_t), investment (i_t), capital utilization cost (z_t) and exogenous spendings (ϵ_t^g):

$$y_t = c_y c_t + i_y i_t + z_y z_t + \epsilon_t^g \tag{13}$$

where the disturbance ϵ_t^g follows a first-order autoregressive process, with an additional component linked to the productivity shock η_t^a ($\epsilon_t^g = \rho_g \epsilon_{t-1}^g + \rho_{ga} \eta_t^a + \eta_t^g$). The inclusion of a productivity innovation is due to the necessity of capturing the effects of productivity in the competitiveness in exterior markets. Parameter c_y represents the consumption-output steady-state ratio, i_y is the investment-output steady-state ratio and $z_y = R^k k_y$, where R^k is the steady-state rental rate of capital.

The monetary policy reaction function

The monetary policy reaction function is described by a the Taylor rule. The interest rate controlled by the policy makers is determined by current inflation, the output gap (defined as the difference between the output under price and wage rigidity and the output under a completely price and wage flexible economy) and the growth rate of the output gap. In addition, the reaction function incorporates a smoothing component which improves the fit to the actual variations in the nominal interest rate. Formally:

$$r_t = \rho r_{t-1} + (1-\rho)[r_\pi \pi_t + r_y(y_t - y_t^p)] + r_{\Delta y}[(y_t - y_t^p) - (y_{t-1} - y_{t-1}^p)] + \epsilon_t^r \tag{14}$$

where the monetary policy disturbance ϵ_t^r is assumed to follow a first-order autoregressive process ($\epsilon_t^r = \rho_r \epsilon_{t-1}^r + \eta_t^r$).

2.1 Alternative specifications

This section introduces alternative specifications we are going to consider in the estimation of the model in order to assess different scenarios which may improve the performance of the model. More precise, we consider survey data on inflation in the estimation of the model, two separated innovations for the persistent and the transitory price (wage) mark-up process, a one- and four-quarter-ahead forward-looking monetary policy rule and an informational asymmetry scenario where policy makers cannot observe actual inflation but forecasted inflation.

Considering the inflation forecasts from SPF in the Smets and Wouters (2007) model

The inflation forecast from SPF may contain additional information about the inflation formation. Mehra (2003) claims that survey data on inflation expectations Granger-cause actual inflation. Consequently, the inclusion of inflation forecast from SPF data in the estimation of the model may help to get a better identification of the transmission mechanism of the shocks. We introduce the SPF as observables following Del Negro and Eusepi (2011). By definition in a rational expectations model agents are able to perfectly anticipate future inflation beyond some small random forecasting errors. Therefore, we force the inflation forecast from the SPF to be equal to inflation plus an IID-Normal error term. The measurement equations are then given by:

$$\pi_{t|t}^{SPF} = \bar{\pi} + \pi_t + \eta_t^{SPF0}$$

$$\pi_{t+1|t}^{SPF} = \bar{\pi} + E_t\pi_{t+1} + \eta_t^{SPF1}$$

$$\pi_{t+4|t}^{SPF} = \bar{\pi} + E_t\pi_{t+4} + \eta_t^{SPF4}$$

where $\pi_{t+i/t}^{SPF}$ denotes the i-quarter-ahead forecasted inflation.

In a following section, we allows the discrepancies between model-based and SPF forecasts to follow a first order autoregressive process in order to consider systematic deviations of the survey forecast from the rational expectations assumption in which the model is based on.

Slobodyan and Wouters (2017) specification for mark-up shocks

The price and wage mark-up shock processes in the Smets and Wouters (2007) model are assumed to follow an ARMA(1,1) process. This specification captures the persistence and the high frequency fluctuations in inflation and wages through a single innovation for each process (η_t^p and η_t^w, respectively). Nevertheless, Slobodyan and Wouters (2017) postulate a specification where the persistence and the high frequency fluctuation components of these shocks are driven by two different innovations. Formally,

$$\epsilon_t^p = \epsilon_{p,t}^{per} + \epsilon_{p,t}^{tran} \qquad \epsilon_t^w = \epsilon_{w,t}^{per} + \epsilon_{w,t}^{tran}$$
$$\epsilon_{p,t}^{per} = \rho_p^{per} \epsilon_{p,t-1}^{per} + \eta_{p,t}^{per} \qquad \epsilon_{w,t}^{per} = \rho_w^{per} \epsilon_{w,t-1}^{per} + \eta_{w,t}^{per}$$
$$\epsilon_{p,t}^{tran} = \eta_{p,t}^{tran} \qquad \epsilon_{w,t}^{tran} = \eta_{w,t}^{tran}$$

The Slobodyan and Wouters (2017) specification can be useful when survey data on inflation is used as an observable in the estimation of the model. The inclusion of this specification is advantageous to the extend that the SPF contains more precise information on inflation expectation formation, which is not captured by other time series included in the set of observables. The consideration of inflation forecasts from the SPF as observables enables the identification of two separate innovation. Thus, innovation $\epsilon_{p,t}^{per}$ captures the persistent part and innovation $\epsilon_{p,t}^{tran}$ captures the high frequency volatility part of the price mark-up shock process. A similar the same specification is considered for the wage mark-up shock process.

Forward-looking monetary policy rule

One may think that nominal interest does not depend on current inflation but on future inflation, because the effects of the change of a monetary policy appear with delay, and policy makers, taking this fact into account, establish nominal interest rate based on future inflation. Hence, we also take into consideration alternative expectational horizons of inflation in the policy reaction-function in order to study the robustness of estimation results. More precisely, we re-estimate the model by considering, as alternatives to current inflation, the one- and four-quarters-ahead inflation expectations(π_{t+1}, π_{t+4}).

The informational asymmetry scenario

We also consider an informational asymmetry scenario where policy makers cannot perfectly anticipate actual inflation expectations and they can only observe what the professional forecasters predict. Indeed, SPF panelists can have a good reputation among policy makers and it is therefore reasonable to assume that policy makers take into account SPF forecasts on inflation to determine the nominal interest rate.

In order to model this informational asymmetry scenario we replace the actual inflation which appears in equation (14) for the forecasted inflation reported in the SPF. In order to compare this informational asymmetry scenario with the symmetric one, we also estimate the model for different expectational horizons, one-quarter-ahead forecasted inflation($\pi^{SPF}_{t+1|t}$) and four-quarter-ahead forecasted inflation ($\pi^{SPF}_{t+4|t}$). Formally,

$$r_t = \rho r_{t-1} + (1-\rho)[r_\pi \pi^{SPF}_{t+i|t} + r_y(y_t - y^p_t)] + r_{\Delta y}[(y_t - y^p_t) - (y_{t-1} - y^p_{t-1})] + \epsilon^r_t$$

where i = 0, 1 and 4.

Persistence discrepancies between model-based and SPF forecasts

There is a large economic literature reviewing the statistical properties of the SPF. The conclusion of many researchers, like Mehra (2003), is that inflation forecasts are non-rational. As shown below, the autocorrelation of inflation forecasts reported in SPF expectations are remarkably higher than the autocorrelation of actual inflation. These findings suggest that professional forecasters are somehow sluggish when processing information and, consequently, inflation forecasts from the SPF does not seem to support the rationality assumption.

Taking this fact into consideration, we allow for persistence in the discrepancy between the inflation forecast reported in the SPF and the corresponding inflation expectation defined in the model:

$$\pi^{SPF}_{t|t} = \bar{\pi} + \pi_t + \epsilon^{SPF0}_t$$
$$\epsilon^{SPF0}_t = \rho^{SPF0} \cdot \epsilon^{SPF0}_{t-1} + \eta^{SPF0}_t$$

$$\pi^{SPF}_{t+1|t} = \bar{\pi} + E_t\pi_{t+1} + \epsilon^{SPF1}_t$$
$$\epsilon^{SPF1}_t = \rho^{SPF1} \cdot \epsilon^{SPF1}_{t-1} + \eta^{SPF1}_t$$

$$\pi^{SPF}_{t+4|t} = \bar{\pi} + E_t\pi_{t+4} + \epsilon^{SPF4}_t$$

$$\epsilon_t^{SPF4} = \rho^{SPF4} \cdot \epsilon_{t-1}^{SPF4} + \eta_t^{SPF4}$$

By implementing this specification we are able to study if the assumption of unpredictable forecasting errors from the SPF is too strong. We study which specification results in a better fit and generates more accurate posterior second moments in order to carry out an model-based test for rationality of the SPF.

3 Estimation

Our sample contains the quarterly data of years 1970:2-2007:4. We consider this start-up year due to the availability of the SPF data on four-quarter-ahead inflation forecast and we do not include years thereafter due to the instability of the time series caused by the economic and financial crisis of 2007.

We consider a Bayesian estimation approach.[1] The Bayes theorem describes the probability of an event A conditional on B as the probability of the event B conditional on A and the marginal probability of A. Therefore, the Bayes theorem allows us to compute the parameters distribution of the model conditional on a set of observables, by computing the probability of obtaining those observables conditional on a certain values of the parameters and the marginal probability of those parameters values. Formally,

$$Prob(\theta_B|Y_t) = \frac{Prob(Y_t|\theta_B) \quad Prob(\theta)}{\int Prob(Y_t, \theta_B) d\theta_B}$$

where θ_B represents the parameters of the model and Y_t represents the observables. Therefore, the posterior distribution of the parameters of the model conditional on the observables depends on a likelihood function, the prior information on the model parameters (numerator) and the marginal probability of the model paremeters values, which is no more than a normalization constant (denominator).

We cannot consider a number of variables greater than the number of shocks in the set of observables, inasmuch as the model would be stochastically singular, invalidating any estimation exercise. Therefore, we need to filter the data in order to

[1] Bayesian estimation techniques are explained in more detail in several papers, for example Fernández-Villaverde (2010)

compute the likelihood function of the observables conditional on the model's parameters. We use the Kalman filter to get the prediction errors of the model for the set of observables and compute the numerical value of the likelihood function from those prediction errors. By maximizing the likelihood function over the parameters we get the modes of these parameters. Nevertheless, due to the complexity of the posterior distribution of the parameters is too difficult to get a complete draw of the parameters distribution. Fortunately, we are interesting in computing certain statistic (ie. mean, standard deviation) which can be obtained by using the Metropolis-Hasting (MH) algorithm. The MH algorithm is no more than a rejection sampling algorithm which allows finding the posterior distribution of a certain statistic of the parameters by only assuming that those statistics are asymptotically normal. In short, what the MH algorithm does is to generate candidates (θ_B^*) from a normal distribution ,$N(\theta_B^{i-1}, \sigma_{mh})$, and evaluates them through the following aceptance rule,

$$\alpha_B = \frac{Prob(\theta_B^*|Y_t)}{Prob(\theta_B^{i-1}|Y_t)}$$

where candidates are accepted if $\alpha_B \geq 1$ and rejected with probability $1-\alpha_B$ if $\alpha_B < 1$.

The set of observables

Smets and Wouters (2007) consider the log differences of real GDP, real consumption, real investment, real wage and the GDP deflator, the log of hours worked and the federal funds rate as observables. In addition to the previous measurement equation for the inflation forecast from SPF, we consider the following measurement equations to introduce these variables in the model:

$$log(GDP_t/GDP_{t-1}) = \bar{\gamma} + y_t - y_{t-1}$$

$$log(CONS_t/CONS_{t-1}) = \bar{\gamma} + c_t - c_{t-1}$$

$$log(INV_t/INV_{t-1}) = \bar{\gamma} + i_t - i_{t-1}$$

$$log(WAGE_t/WAGE_{t-1}) = \bar{\gamma} + w_t - w_{t-1}$$

$$log(GDPdef_t/GDPdef_{t-1}) = \bar{\pi} + \pi_t$$

$$log(HOURS_t) = \bar{l} + l_t$$

$$FEDFUNDS_t = \bar{r} + r_t$$

where $\bar{\gamma}$ is the exogenous quarterly growth rate trend, $\bar{\pi}$ is the quarterly steady-state inflation rate, $\bar{r}$ is the quarterly steady-state nominal interest rate and $\bar{l}$ is the steady-state hours worked which is normalized to 0.

Prior distribution of the parameters

We consider the prior distribution of the parameters described in Smets and Wouters (2007). In addition to the original model parameters, we have included certain parameters and innovations whose prior distributions are chosen consistently with Smets and Wouters (2007) choice. The prior distribution of the additional innovations (persistent and transitory wage and price mark-up innovations and the innovations which featuring the discrepancy between the rational expectation of inflation derived from the model and the corresponding inflation forecast from the SPF) are assumed to follow a inverse-gamma distribution with means 0.1 and 2 degrees of freedom. The persistence of the AR(1) processes for the SPF measurement errors, wage and price mark-ups are assumed to follow a beta distribution with means 0.5 and standard deviation 0.2.

4 Model comparison

Table 3 shows the marginal likelihood of the alternative specifications we have estimated. We are considering the Slobodyan and Wouters (2017) specification in models 3 to 8. We investigate whether the consideration of informational asymmetry for policy makers improves the fit of the estimated models. In this line, models 6 to 8 do not the inflation but the forecasted inflation from the SPF incorporate in the monetary policy reaction function. Moreover, we carry out a robustness exercise for alternative policy rule specifications with respect to the expectational horizon of inflation. In order to perform this analysis we assume a forward-looking policy rule with one quarter-ahead inflation expectations in models 4 and 7, and with four-quarter-ahead inflation expectations in models 5 and 8. Finally, the second row shows the marginal likelihood of the alternative specification when we allow for persistence in the discrepancy between the rational expectation of inflation derived from the model and the corresponding inflation forecast from the SPF.

Table 1: Comparison of the Marginal Likelihood of alternative specification.

	Smets & Wouters 7obs	Smets & Wouters 10 obs	Model 3 10 obs	Model 4 10 obs	Model 5 10 obs	Model 6 10 obs	Model 7 10 obs	Model 8 10 obs
WN measurement error	-845.29	-789.54	-693.95	-689.91	-697.05	-706.28	-705.78	-708.37
AR(1) measurement error			-667.77	-666.96	-673.36	-685.07	-683.94	-692.23

It is noticeable that the Slobodyan and Wouters (2017) specification achieves a large improvement in the marginal likelihood, about 95 points. When we consider informational asymmetry, the fit of the model worsen about 13 points. On the other hand, assuming that the discrepancy between the model inflation expectation and the forecasted inflation from the SPF follows a first-order autoregressive process, we get an improvement in the fit of roughly 23 points for the best fitting model (Model 4). This result raises doubts about whether professional forecasters are perfectly rational and whether the rational expectations model can accurately replicate the observed data of inflation expectations. Alternatively, the finding of a persistence discrepancy between model-based and SPF inflation forecasts shows that SPF inflation forecast provide distinctive observable data on inflation expectations which might be important in assessing the importance of alternative sources of persistence.

We conclude that Model 4' (with an slight advantage with regards to Model 3') is the specification which presents the best fit.

Estimation results[2]

Table 1 shows the estimation of the structural parameters for alternative specifications. The second column shows the Smets and Wouters (2007) model estimation including the SPF as observables. The third column shows the Smets and Wouters model estimation considering the Slobodyan and Wouters (2017) specifi-

[2]We also estimate the informational asymmetry models, finding rather robust results with respect to the complete information model. Furthermore, the marginal likelihood analysis shown in section 4 concludes that these models have a worse fit than the complete information models. Results are available under request.

cation (Model 3). The fifth column shows the estimation of the model where we consider a forward-looking policy rule (Model 4). The fourth and sixth columns contain the analogous estimation to models 3 and 4, respectively, but considering a persistence discrepancy between model-based and SPF inflation forecasts.

Overall, the estimation of the parameters across the alternative specifications are rather robust. Nevertheless, the variation of price and wage rigidities across the estimation of the alternative models should be mentioned. The Calvo probability for wages increases when we force the discrepancy between the inflation forecast from the SPF and the model inflation expectation to be an unpredictable white noise error. Nevertheless, when we relax this assumption by supposing a persistent process, the Calvo probability for wages decreases with respect to the Smets and Wouter (2007) model. The Calvo probability which determined the price stickiness increases when we consider SPF as observables and this rise becomes more noticeable if we assume a persistent discrepancy between model-based and SPF inflation forecasts. Moreover, the indexation coefficients of past inflation in the New-Keynesian Phillips curve decreases remarkably from 0.30 to 0.05 when we consider a persistent discrepancy between model-based and SPF inflation forecasts. The indexation of past inflation in wages also reduces its weight considerably if we allow for a persistent discrepancy between the model-based and SPF inflation forecasts.

Table 2 presents the estimation of the standard deviations and persistent parameters of the shock processes. The estimations are considerably robust across the alternative specifications, with the exception of the standard deviation of the innovation which captures the high frequency volatility of the wage mark-up process which decreases when we consider a persistent discrepancy between model-based and SPF inflation forecasts. Whereas, by considering this persistent discrepancy, the AR(1) parameter of the price mark-up process undergoes an increase. The decrease of the persistence of the risk premium shock when we introduce a persistence structure in this discrepancy should be also mentioned .

It is also noticeable that the parameters which capture the persistence of this discrepancy increase when we look at the one and four-quarters-ahead inflation forecast. The high values of the autocorrelation parameters reflect the highly inertia associated with SPF inflation forecasts. Besides, this high inertia suggests professional forecasters do not employ all relevant information in generating their inflation prediction. Therefore, the survey data on inflation expectations from SPF seems to violate the rationality assumption.

Table 2: Comparison of the Posterior distribution of Structural Paramenters of alternative specifications.

	Smets & Wouters 7 obs	Smet & Wouters 10 obs	Model 3 10 obs	Model 3' 10 obs	Model 4 10 obs	Model4' 10 obs
φ	5,12	3.53	4,39	4,65	4,81	4,73
	(3,40 6,75)	(2,00 5,40)	(2,80 5,96)	(3,00 6,20)	(3,19 6,42)	(3,07 6,33)
σ_c	1,46	1,75	1,50	1,51	1,51	1,48
	(1,21 1,70)	(1,38 2,18)	(1,23 1,80)	(1,26 1,77)	(1,26 1,75)	(1,23 1,71)
h	0,68	0,54	0,66	0,65	0,67	0,66
	(0,60 0,76)	(0,42 0,66)	(0,57 0,75)	(0,57 0,72)	(0,59 0,76)	(0,58 0,75)
ξ_w	0,72	0,83	0,76	0,65	0,75	0,64
	(0,61 0,83)	(0,80 0,87)	(0,72 0,80)	(0,58 0,72)	(0,71 0,80)	(0,58 0,72)
σ_l	1,75	1,75	2,13	1,29	2,28	1,36
	(0,68 2,72)	(1,37 2,18)	(1,16 3,06)	(0,43 2,11)	(1,52 3,27)	(0,46 2,21)
ξ_p	0,68	0,82	0,78	0,80	0,75	0,78
	(0,60 0,76)	(0,76 0,87)	(0,72 0,84)	(0,74 0,86)	(0,68 0,82)	(0,73 0,84)
ι_w	0,57	0,50	0,65	0,39	0,63	0,40
	(0,35 0,78)	(0,30 0,69)	(0,46 0,85)	(0,18 0,60)	(0,44 0,84)	(0,18 0,61)
ι_p	0,30	0,1	0,09	0,05	0,10	0,05
	(0,11 0,47)	(0,04 0,16)	(0,03 0,14)	(0,01 0,08)	(0,04 0,15)	(0,02 0,08)
ψ	0,63	0,68	0,64	0,61	0,64	0,61
	(0,48 0,78)	(0,52 0,84)	(0,49 0,80)	(0,46 0,76)	(0,51 0,78)	(0,46 0,77)
Φ	1,71	1,66	1,71	1,72	1,75	1,71
	(1,59 1,84)	(1,53 1,78)	(1,57 1,86)	(1,59 1,85)	(1,61 1,90)	(1,59 1,84)
r_π	2,03	1,81	2,04	2,00	2,13	2,14
	(1,74 2,32)	(1,54 2,09)	(1,75 2,32)	(1,72 2,27)	(1,87 2,40)	(1,85 2,43)
ρ	0,81	0,81	0,84	0,82	0,78	0,78
	(0,77 0,85)	(0,76 0,85)	(0,80 0,87)	(0,78 0,86)	(0,73 0,83)	(0,73 0,83)
r_y	0,09	0,11	0,13	0,12	0,13	0,11
	(0,05 0,13)	(0,07 0,15)	(0,09 0,17)	(0,08 0,15)	(0,10 0,17)	(0,08 0,15)
$r_{\Delta y}$	0,23	0,24	0,24	0,24	0,20	0,22
	(0,18 0,27)	(0,19 0,29)	(0,19 0,28)	(0,19 0,29)	(0,15 0,24)	(0,17 0,27)
$\bar{\pi}$	0,65	0,63	0,61	0,63	0,60	0,64
	(0,54 0,76)	(0,51 0,76)	(0,49 0,73)	(0,52 0,74)	(0,49 0,71)	(0,53 0,75)
$100(\beta^{-1}-1)$	0,25	0,26	0,25	0,25	0,25	0,25
	(0,09 0,40)	(0,09 0,41)	(0,09 0,40)	(0,09 0,41)	(0,09 0,40)	(0,09 0,41)
$\bar{l}$	1,31	2.04	2,54	1,87	3,24	1.89
	(-0,18 2,76)	(0,35 3,73)	(0.96 4,09)	(0,49 3,30)	(1,80 4,69)	(0,50 3,32)
$\bar{\gamma}$	0,43	0,43	0,43	0,44	0,42	0,44
	(0,40 0,46)	(0,40 0,47)	(0,39 0,46)	(0,41 0,47)	(0,38 0,46)	(0,41 0,46)
α	0,29	0,23	0,29	0,29	0,32	0,28
	(0,22 0,35)	(0,16 0,30)	(0,21 0,37)	(0,22 0,36)	(0,25 0,40)	(0,22 0,35)

Table 3: Comparison of the Posterior distribution of Shock Processes of alternative specifications.

	Smets & Wouters 7 obs	Smets & Wouters 10 obs	Model 3 10 obs	Model 3′ 10 obs	Model 4 10 obs	Model 4′ 10 obs
σ_{SPF0}		0,24	0,24	0,23	0,24	0,23
		(0,22 0,27)	(0,22 0,27)	(0,21 0,25)	(0,22 0,27)	(0,21 0,26)
σ_{SPF1}		0,15	0,07	0,09	0,08	0,09
		(0,14 0,17)	(0,05 0,09)	(0,08 0,11)	(0,06 0,09)	(0,07 0,11)
σ_{SPF4}		0,14	0,06	0,10	0,06	0,09
		(0,13 0,16)	(0,05 0,07)	(0,08 0,11)	(0,05 0,07)	(0,09 0,11)
σ_a	0,39	0,40	0,39	0,39	0,38	0,39
	(0,35 0,43)	(0,36 0,45)	(0,34 0,43)	(0,35 0,43)	(0,34 0,42)	(0,35 0,43)
σ_b	0,23	0,18	0,21	0,23	0,23	0,24
	(0,19 0,27)	(0,12 0,23)	(0,16 0,27)	(0,19 0,27)	(0,18 0,27)	(0,20 0,27)
σ_g	0,49	0,49	0,50	0,50	0,50	0,50
	(0,44 0,54)	(0,44 0,54)	(0,45 0,55)	(0,45 0,55)	(0,45 0,55)	(0,45 0,55)
σ_i	0,42	0,51	0,46	0,45	0,46	0,45
	(0,35 0,49)	(0,34 0,75)	(0,38 0,53)	(0,37 0,52)	(0,38 0,53)	(0,37 0,52)
σ_r	0,25	0,25	0,25	0,25	0,24	0,25
	(0,22 0,27)	(0,22 0,27)	(0,22 0,27)	(0,22 0,28)	(0,21 0,26)	(0,22 0,28)
$\sigma_p/\ \sigma_p^{per}$	0,13	0,18	0,05	0,03	0,04	0,03
	(0,10 0,16)	(0,15 0,20)	(0,03 0,06)	(0,22 0,04)	(0,03 0,06)	(0,02 0,04)
σ_w/σ_w^{per}	0,30	0,42	0,02	0,03	0,02	0,03
	(0,25 0,34)	(0,37 0,47)	(0,01 0,02)	(0,02 0,04)	(0,01 0,02)	(0,02 0,04)
μ_p/σ_p^{trans}	0,63	0,79	0,22	0,20	0,22	0,19
	(0,44 0,83)	(0,73 0,84)	(0,19 0,25)	(0,17 0,22)	(0,20 0,25)	(0,17 0,22)
μ_w/σ_w^{trans}	0,87	0,99	0,46	0,36	0,46	0,35
	(0,78 0,96)	(0,99 0,99)	(0,41 0,51)	(0,31 0,40)	(0,40 0,51)	(0,31 0,39)
ρ_a	0,95	0,95	0,95	0,95	0,95	0,95
	(0,93 0,96)	(0,93 0,98)	(0,93 0,97)	(0,93 0,97)	(0,93 0,97)	(0,93 0,97)
ρ_b	0,22	0,47	0,33	0,21	0,27	0,20
	(0,08 0,36)	(0,22 0,73)	(0,09 0,56)	(0,07 0,36)	(0,09 0,43)	(0,06 0,34)
ρ_g	0,96	0,97	0,97	0,96	0,97	0,96
	(0,94 0,98)	(0,96 0,99)	(0,95 0,98)	(0,94 0,98)	(0,95 0,98)	(0,94 0,98)
ρ_i	0,75	0,83	0,69	0,70	0,69	0,70
	(0,65 0,87)	(0,66 1,00)	(0,59 0,80)	(0,60 0,80)	(0,59 0,79)	(0,59 0,80)
ρ_r	0,15	0,12	0,11	0,13	0,16	0,18
	(0,04 0,25)	(0,03 0,20)	(0,02 0,18)	(0,03 0,21)	(0,04 0,27)	(0,06 0,30)
ρ_{ga}	0,61	0,58	0,61	0,61	0,61	0,60
	(0,23 0,77)	(0,43 0,75)	(0,44 0,77)	(0,45 0,79)	(0,44 0,78)	(0,43 0,77)
$\rho_p/\ \rho_p^{per}$	0,83	0,86	0,68	0,79	0,71	0,81
	(0,71 0,94)	(0,74 0,94)	(0,59 0,77)	(0,73 0,85)	(0,61 0,80)	(0,75 0,87)
ρ_w/ρ_w^{per}	0,96	1,00	0,98	0,98	0,98	0,98
	(0,94 0,99)	(0,99 1,00)	(0,97 0,99)	(0,97 0,99)	(0,97 0,99)	(0,97 0,99)
ρ_{SPF0}^{per}				0,32		0,32
				(0,20 0,44)		(0,20 0,44)
ρ_{SPF1}^{per}				0,83		0,78
				(0,73 0,92)		(0,65 0,93)
ρ_{SPF4}^{per}				0,87		0,87
				(0,81 0,94)		(0,80 0,93)

Note: Those rows where two different parameters appear separated by a bar shows the estimation of the parameter on the left hand-side for the SW model with 7 and 10 obs and the estimation of the parameter on the right hand-side for the rest of the models

Posterior Second Moments analysis

Table 4 shows the second moments of the actual data and the posterior second moments estimated from alternative specifications for the whole set of observables. Regarding the volatility, the models where we allow for persistence discrepancies between model-based and SPF inflation forecasts reproduce much better the size of the fluctuations of forecasted inflation and the growth of the real wages and investment, whereas the models with unpredictable white noise discrepancies perform much better in reproducing the nominal interest rate volatility. Moreover both specifications performs closely reproducing the growth rate of output and consumption.

Regarding the contemporaneous correlation with output growth, models 3' and 4' slightly outperforms models 3 and 4 for the growth of consumption, investment and labor. Moreover, models 3' and 4' are much better in reproducing this correlation for the nominal interest rate, whereas the opposite case is true when we look at the growth rate of real wages. Moreover, it is noticeable that all the specifications do a great job in reproducing correlation with output growth for the inflation forecasts.

Regarding the contemporaneous correlation with inflation, all specifications perform closely in reproducing this co-movements for the growth rate of output, real wages and consumption, whereas models 3' and 4' present a slightly advantage with respect to the nominal interest rate and the inflation forecasts. Concerning the growth rate of labor, models where we consider a persistent discrepancy between model-based and SPF inflation forecasts overestimated its co-movements with inflation, whereas models 3 and 4 underestimate this correlation with a similar error.

Concerning to the first-order autocorrelation we observe that the alternative specification perform closely in reproducing all the variables autocorrelation. It is also noticeable the difficulty of replicating the high autocorrelation of the survey data on inflation not only by Model 4' but by all the models. This is also a signal of the irrationality of the SPF inflation forecast whose autocorrelation coefficients cannot be accurately reproduced by a rational expectations model.

Finally, having analysed the posterior second moments statistics of the alternative specifications we can conclude that Model 4' is the best in describing these second moments statistics with a slight advantage with respect to Model 3'.

Table 4: Comparison of the Posterior Second Moments.

		Actual data	Smets & Wouters 7obs	Smets & wouters 10obs	Model 3 10obs	Model 3′ 10obs	Model 4 10obs	Model 4′ 10obs
Output growth	Volatility	0,71	0,98	0,98	0,85	0,87	0,79	0,85
	Corr with output	1,00	1,00	1,00	1,00	1,00	1,00	1,00
	Corr with inflation	-0,30	-0,19	-0,06	-0,12	-0,08	-0,06	-0,07
	Autocorr of order 1	0,24	0,14	0,21	0,20	0,19	0,19	0,20
Consumption growth	Volatility	0,46	0,53	0,49	0,42	0,43	0,36	0,41
	Corr with output	0,65	0,63	0,61	0,60	0,62	0,59	0,61
	Corr with inflation	-0,40	-0,25	-0,08	-0,18	-0,13	-0,10	-0,12
	Autocorr of order 1	0,20	0,15	0,27	0,29	0,18	0,27	0,24
investment growth	Volatility	4,18	4,49	4,12	3,88	4,38	3,38	4,07
	Corr with output	0,70	0,67	0,68	0,66	0,68	0,66	0,67
	Corr with inflation	-0,20	-0,12	-0,03	-0,06	-0,03	-0,01	-0,03
	Autocorr of order 1	0,54	0,29	0,46	0,41	0,41	0,41	0,42
labor growth	Volatility	7,86	7,82	11,37	13,03	7,24	7,76	6.77
	Corr with output	0,11	0,15	0,10	0,10	0,14	0,10	0,11
	Corr with inflation	-0,52	-0,27	-0,49	-0,66	-0,46	-0,57	-0,50
	Autocorr of order 1	0,97	0,47	0,73	0,74	0,74	0,73	0,73
Inflation	Volatility	0,38	0,32	0,37	0,53	0,26	0,34	0,29
	Corr with output	-0,30	-0,19	-0,06	-0,12	-0,08	-0,06	-0,07
	Corr with inflation	1,00	1,00	1,00	1,00	1,00	1,00	1,00
	Autocorr of order 1	0,90	0,41	0,64	0,62	0,56	0,58	0,61
Wage growth	Volatility	0,41	0,42	0,92	0,75	0,57	0,74	0,50
	Corr with output	0,04	0,28	0,15	0,19	0,28	0,18	0,28
	Corr with inflation	-0,19	-0,16	-0,06	-0,08	-0,10	-0,09	-0,12
	Autocorr of order 1	0,04	0,10	-0,01	0,01	0,06	-0,02	0,07
interest rate	Volatility	0,74	0,42	0,59	0,61	0,36	0,43	0,34
	Corr with output	-0,21	-0,16	-0,03	-0,11	-0,10	-0,08	-0,14
	Corr with inflation	0,62	0,65	0,74	0,78	0,64	0,70	0,59
	Autocorr of order 1	0,95	0,44	0,70	0,70	0,67	0,70	0,67
Nowcast	Volatility	0,34		0,44	0,61	0,29	0,41	0,28
	Corr with output	-0,27		-0,06	-0,11	-0,08	-0,05	-0,06
	Corr with inflation	0,92		0,92	0,94	0,91	0,91	0,91
	Autocorr of order 1	0,96		0,54	0,54	0,50	0,47	0,51
One-quarter-ahead inflation forecast	Volatility	0,28		0,34	0,44	0,21	0,26	0,18
	Corr with output	-0,24		-0,03	-0,07	-0,04	-0,05	-0,05
	Corr with inflation	0,89		0,92	0,93	0,85	0,90	0,89
	Autocorr of order 1	0,97		0,65	0,71	0,69	0,69	0,70
Four-quarter-ahead inflation forecast	Volatility	0,21		0,29	0,38	0,19	0,22	0,15
	Corr with output	-0,14		-0,01	-0,05	-0,02	-0,03	-0,02
	Corr with inflation	0,79		0,90	0,89	0,74	0,86	0,76
	Autocorr of order 1	0,97		0,68	0,71	0,71	0,72	0,70

Variance decomposition and IRF

Having analysed how models perform in reproducing the main US business cycle features, we focus now on the variance decomposition of shocks, which assess the importance of alternative sources of fluctuations. Figure 2 shows the forecast error variance decomposition of inflation, interest rate and four-quarter-ahead inflation forecast at various horizons. The graphs on the left-hand side correspond to the variance decomposition of Model 4' whereas the right-hand side graphs show the decomposition of Model 4.[3]

By allowing for persistence in the discrepancy between the model inflation expectation and the forecasted inflation from the SPF (Model 4'), the relative importance of the persistent part of the wage mark-up shock decreases substantially in favour of different shocks across the different horizons. The reduction of the effect of this shock is also noticeable in the impulse-response functions shown in Figure 1.

By considering this persistent discrepancy between model-based and SPF inflation forecasts, we observe an increase of the relative importance of the persistent price mark-up shock as a driven force of the inflation fluctuations. Moreover, this weight gain becomes more remarkable for distant horizons. The importance gain of this shock is in detriment of the persistent wage mark-up shock which, in Model 4, drives most of the medium- and long-run fluctuation of inflation (65%), whereas, in Model 4', an important part of the volatility is due to the persistent (25%) and transitory (24%) price mark-up. Moreover, it should be mentioned the negligible effects of the transitory wage mark-up (more remarkable in Model 4') in the volatility of inflation for the different horizons. In addition, the effect of the transitory wage mark-up shock is notoriously short-lived. This can also be seen in Figure 1 where, in the impulse-response function of inflation to the transitory wage mark-up shock, there is a positive effect which disappears in the second quarter and is, consequently, invisible in the impulse-response function of the one quarter-ahead forecasted inflation.

Regarding the nominal interest rate variance decomposition, we can also observe a reduction of the relative importance of the persistent wage mark-up in the

[3] We have also looked at the differences across different specifications in terms of expectational horizons and informational asymmetry finding no remarkable differences.

medium- and long-run. The reduction of the weight of this shock imply an increase of the importance of the monetary policy shock in determining the volatility of the nominal interest rate. Moreover, it is also noticeable that both specifications allows us to identify a larger importance in the medium- and long-run of the risk premium shock than the importance that Smets and Wouters (2007) find.

Finally, the reduction of the persistent wage mark-up from Model 4 to Model 4' is specially remarkable in the variance decomposition of the inflation forecast. In the case of inflation forecast, this reduction implies a considerable weight gain of the persistent price mark-up shock and the discrepancy between model-based and SPF inflation forecasts. The weight gain of this discrepancy may indicate the non-efficiency of the inflation forecast from SPF, arising doubts about their rationality. Similar results rise from the variance decomposition of the nowcast and one-quarter-ahead inflation forecast.

Figure 1: IRF for iid and AR price markup, iid and AR wage markup, and productivity shocks

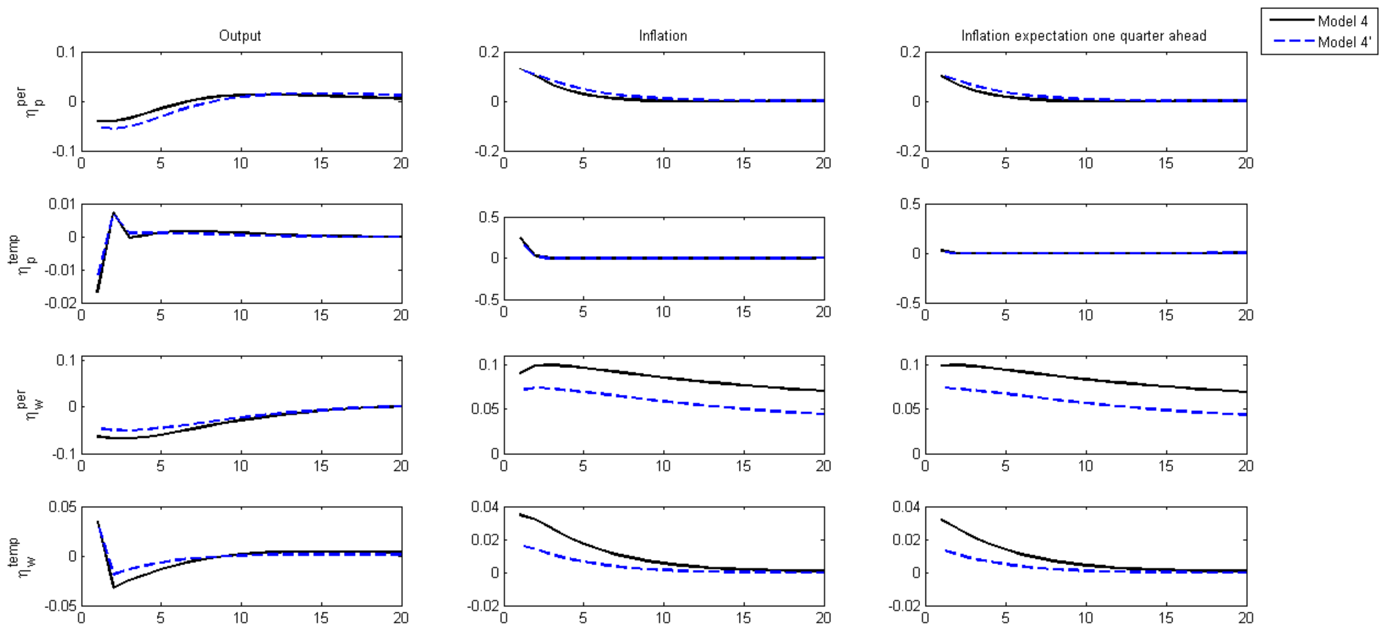
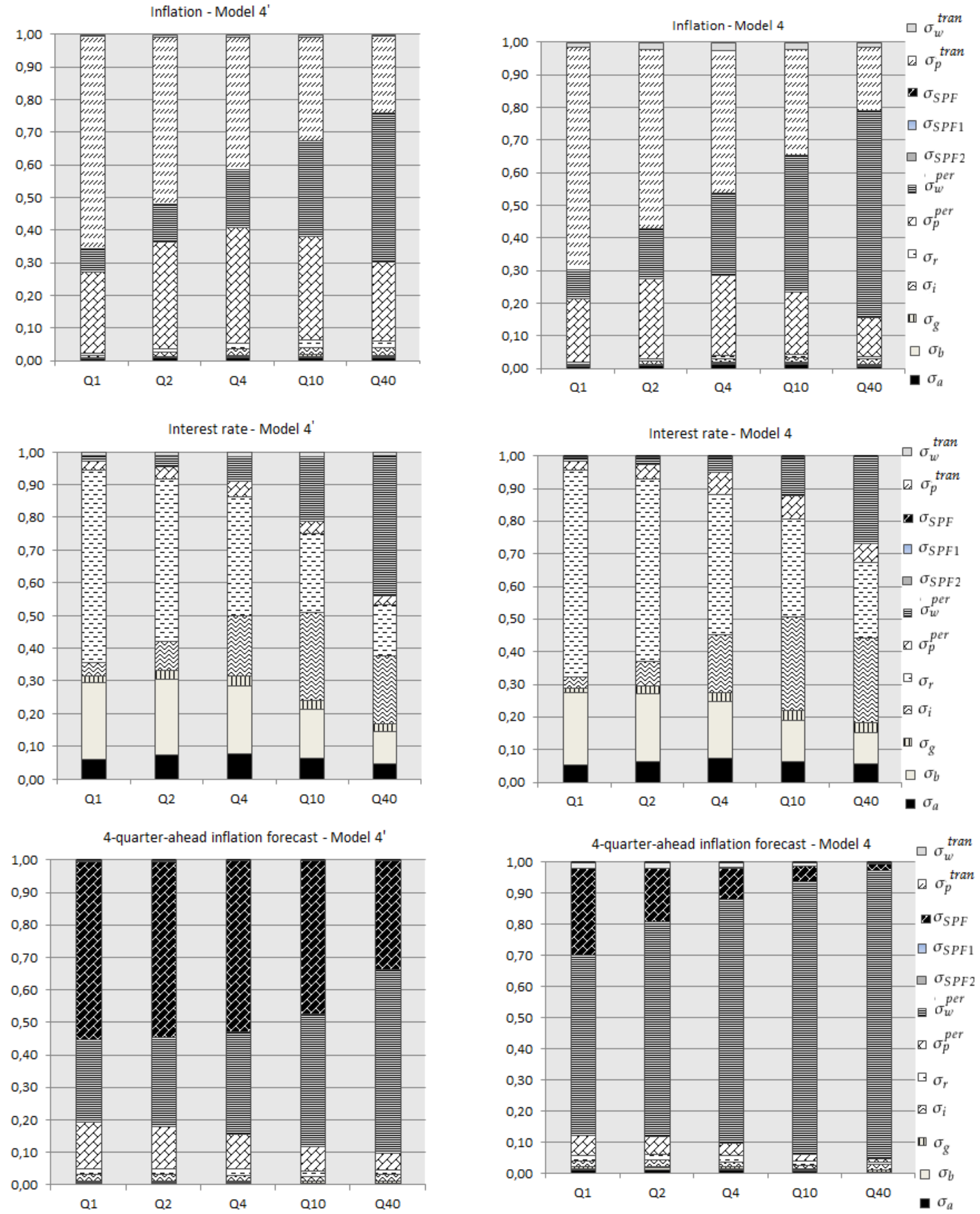


Figure 2: Variance Decomposition of variables genereted by Model 4 and Model 4’



Empirical importance of alternative sources of rigidities

Having analysed the changes in the effects of the different shocks on the variations of the model variables and concluding that the wage mark-up shock effects have been reduced, we are going to focus on the empirical importance of some key parameters of the Smets and Wouters (2007) model.

Table 5: Assessment of the empirical importance of the rigidities from a Marginal likelihood approach.

	Baseline	Marginal Likelihood Losses							
		$\xi_p = 0.1$	$\xi_w = 0.1$	$\iota_p = 0$	$\iota_w = 0$	$\psi = 0.99$	$\psi = 0.01$	$h = 0.1$	$\varphi = 0.1$
Model 4'	-666.96	122	74	-13	-1	18	183	55	119

Table 5 shows the penalty in the marginal likelihood when we reduce the importance of a certain parameter. Smets and Wouters (2007) find that the fact of reducing the importance of price and wage rigidities is equally penalized in terms of marginal likelihood. Nevertheless, we find that if we include survey data on inflation in the estimation, in the way we have mentioned in the previous section, price stickiness seems to be more important than wage stickiness but the latter is still important. In reference to the price and wage indexation, we get similar results as Smets and Wouters (2007), which are not empirically important. As a matter of fact, the consideration of the wage indexation significantly worsen the fit of the model. The capital adjustment cost and the habit formation are rather important. Smets and Wouters (2007) find that the consideration of variable capital utilization has no empirical importance in the estimation of the model. Nevertheless, by including survey data on inflation expectations, like it is done in Model 4', we find that assuming constant capital utilization ($\psi = 0.99$) imply a penalty in the marginal likelihood. Moreover, the consideration the opposite scenario imply a larger penalty of roughly 183 points. Therefore, we conclude that the consideration of variable capital utilization in the estimation of the model is crucial.

5 Conclusion

This paper analyses the consequences of including SPF inflation forecasts as observables, through different specifications, in the estimation of the Smets and Wouters (2007) DSGE model. A remarkable improvement in model fit is obtained by distinguishing the innovations affecting the permanent and transitory shocks of price- and wage- markup shocks as suggested in the Slobodyan and Wouters (2017). This specification results in a better identification of the persistent and the transitory shocks in both actual and forecasted inflation. Another noticeable improvement is produced by allowing a persistent discrepancy between model-based inflation and SPF inflation forecasts. This specification results in an overall better identification of the shock. In particular, we find that the persistence of wage mark-up shocks reduces remarkably its weight in explaining the fluctuations of inflation, interest rates and inflation expectations. Moreover, the persistent and transitory price mark-up becomes more significant explaining actual and forecasted inflation fluctuations. Moreover, the the discrepancy between model-based inflation and SPF inflation forecasts becomes more remarkable in explaining the fluctuation of inflation forecasts.

Model fit improvement is also noticeable when looking at the ability of the model to reproduce second moment statistics of the observed variables. Thus, a specification allowing for persistent discrepancy between model-based and SPF inflation forecasts outperforms both the Smets and Wouters (2007) and the Slobodyan and Wouters (2017) specifications. The most remarkable differences obtained from this persistent discrepancy are the reduction of the Calvo probability for wages and the reduction of the indexation of past inflation in the determination of wages, reducing overall wage nominal rigidity.

Christiano, Eichembaun and Evans (2005) claim that including wage rigidities the inclusion of price rigidities becomes redundant. Nevertheless, Smets and Wouters (2007) show that the deterioration in the marginal likelihood produced by reducing the wage and the price rigidity is equally remarkable. In this paper we show that taking into consideration survey data on inflation price stickiness becomes even more important in terms of model fit than wage rigidities.

Once we allow for a persistent discrepancy between the model inflation expectation and the forecasted inflation from the SPF, we assume that professional forecasters do not consider all the available information in period t in order to predict future inflation, i.e. we assume that SPF's inflation forecast does not support the rational expectation assumption. Therefore we can consider that by relaxing the assumption that SPF's inflation forecasts are consistent with the rational expectations on inflation assumed by the model and looking at the consequences of this assumption we are carrying out a indirect-test for rationality of the SPF. Models where we consider a persistent discrepancy between model-based inflation and SPF inflation forecasts, clearly outperform the rest of the models in terms of fit and reproducing the second moments of actual data. Regarding these results, we conclude that SPF does not support the rational expectations assumption.

Concerning future researches it would be interesting to carry out a similar analysis taking into account additional survey data such as on consumption, investment and other macroeconomic forecasts. One then may assess the performance of the model by allowing for non-rational departures of these survey forecasts and evaluate the relevance of the additional information that these survey data may contain.

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