MODELLING AND FORECASTING REALIZED VOLATILITY IN THE INTRADAY CONTINUOUS GERMAN-AUSTRIAN ELECTRICITY MARKET

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Abstract

There is growing interest in modelling the dynamics of electricity prices. A better understanding of the electricity prices dynamics means better forecasts increasing the competitiveness in the market which is interesting for all the participants in the market. However, the electricity prices present some features such as high variation which make difficult their modelling. In this paper, as there are not good enough models that can predict the exact price of the electricity, we model the realized volatility (RV) so that predictions on the variation can be made. In this sense, the quadratic variation theory is used to model the RV. The RV is separated into the jump component of the variation and the continuous component, then if necessary GARCH structures are included in the models. The intraday continuous market in Germany and Austria for 15-minutes blocks is analysed, this market is inside the EPEX market. It is shown that better results are achieved when the RV is decomposed, even if GARCH structures are included in the models. According to out-of-sample criteria, The best forecasts in this market are achieved using the HAR-CV-JV model.

1 Introduction

A better understanding of the electricity prices dynamics is very important. As German and Ronconi (2006) note electricity price risk forces the energy industry to more developed forecasting models. In particular, the analysis of volatility is crucial to ensure market efficiency and derivative pricing. The idea that volatility estimates based on high frequency data, also called realized volatility (RV), improves our understanding of the electricity prices dynamics was introduced by Barndorff-Nielsen and Shephard (BNS) (2004). The paper follows Andersen, Bollerslev and Diebold (ABD) (2007) paper to analyse whether the forecasts of the RV are better when separating it into jump and non-jump components and two non linear transformations. Andersen, Bollerslev, Diebold and Labys (2003) suggest that simple models of RV explain better electricity prices dynamics than GARCH and related stochastic volatility models in out-of-sample forecasting. Jiang (1999) notes that the conditional volatility and jump size and frequency must be estimated as latent variables. Recent developments in the variation analysis show that the partition in jump and non-jump components of the variation is possible and that the discontinuous jump component can be calculated as the difference between RV and bipower variation.
Electricity is a non storable good, therefore prices are really volatiles. Price spikes due to supply and demand shocks are very important in electricity markets. Hence with high frequency data, it is easy to apply RV techniques to electricity prices. The quadratic variation approach is employed to identify significant price jumps, the estimates of jump and non-jump components are made using non-parametric estimates introduced by BNS (2004), because the prevalence of jumps is very high in energy markets and parametric approaches are not efficient enough. The main difference between electricity markets and financial markets is that the electricity prices follow patterns depending on the hour of the day, day of the week and month of the year, consequently, non zero mean prices must be taken into account. It has been suggested by Ullrich (2012) and ABD (2007) that in the jump detection larger lags must be considered comparing to the lags assumed in Chang et al (2008).
The forecasts are made using heterogeneous autoregressive processes (HAR) introduced by Muller et al, (1990) and Corsi (2004) in which RV is parametrized as a function of its own lags, so called HAR-RV models. This HAR-RV model was improved by ABD (2007) introducing the jump component of the variation, so called HAR-RV-CJ forecasting model. Chang et al, (2008) made a modification in the HAR-RV-CJ model, separating the RV into the jump component and continuous path of the variation, i.e. HAR-CV-JV model.
In this paper two models are estimated to forecast volatility in the intraday continuous 15-minutes contracts German and Austrian market. The first considering only recent past observations of RV. The second separates the RV into the jump component and the continuous path. Following ABD (2007) two non linear transformations of the RV are considered, the logarithmic and the square root transformations, in both models. Finally, GARCH structures are included in the models if necessary. Therefore, after estimating the models, the models are tested and the best predictions are chosen using different criteria.
This paper is mainly based on the papers Chan et al (2008) and Ullrich (2012). The Chang et al (2008) applies the models mentioned above in the Australian electricity mar-

ket, but considering only one lag in the jump detection. In this paper instead we consider lags from 0 to 7, as suggested by Ullrich (2012), an then the number of lags that fits better the data is chosen, Ullrich (2012) affirms that in electricity prices if only one lag is considered the jump detection could be downward bias. The Ullrich (2012) is a paper where a proper jump detection is suggested and the Australian and American electricity markets are compared but only concerning about descriptive statistics without calculating the models. In this paper we go a step further and we include GARCH and EGARCH structures in the models mentioned above. The aim of this paper is to conclude which model makes the best forecasts of the RV in the intraday continuous 15-minute blocks German and Austrian market, therefore high frequency data are used. As expected, for any participant in the market any improvement in the volatility forecasts is beneficial.
There have been done many research in Australian and United States electricity markets, we would like to check how the models developed for those countries work in Europe, precisely in two important countries in European countries, Germany and Austria, where a big share of the whole electricity in Europe is traded.
The rest of the paper is organized as follows. Section 2 explains how to use the quadratic variation theory to decompose the RV, in Section 3 the market we analyse is introduced and some interesting descriptive statics on the data are shown together with some relevant graphs. In Section 5 the models we are going to use are introduced with its estimations, it provides also a brief introduction to GARCH structures. Consequently, in Section 6 the forecasts of different models are compared using different criteria, showing the best model to forecast realized volatility in this market. Finally, Section 7 provides the most important conclusions.

2 Realized volatility and jump detection

This section shows how to calculate the RV based on the quadratic variation theory (QVT). The detailed development of the calculations can be found in BNS (2004,2006) and ABD (2007).
Let $p(t)$ denote the electricity price at time t and assume the price process follows a continuous-time stochastic-volatility model with an additive jump component:

$$dp(t) = \mu(t) + \sigma(t)dW(t) + \kappa(t)dq(t), \tag{1}$$

where $q(t)$ is a Poisson counting process, with $dq(t) = 1$ if there is a jump at time t and zero otherwise. $\kappa(t)$ represents the corresponding jump size at time t if $dq(t) = 1$. $\mu(t)$ and $\sigma(t)$ are the drift and instantaneous volatility respectively. $W(t)$ is a standard Wiener process. The quadratic variation (QV) at time t of the cumulative return process is calculated as,

$$QV_t = \int_{t-1}^{t} \sigma^2(s)ds + \sum_{s=0}^{q(t)} \kappa^2(s). \tag{2}$$

Note that when $q(t) = 0$, i.e. there is no jump, the summation term from Equation (2) disappears. If there is no jump the QV is equal to the integrated volatility (IV) of the continuous sample path process,

$$IV_t = \int_{t-1}^{t} \sigma^2(s)ds. \tag{3}$$

Observe how if the jumps are small enough the QV does not differ too much from the IV. BNS(2004, 2006) propose a statistical test to detect significant or not significant difference between these two equations, that is, to check whether the jumps are significant or not, using the previous notation, whether $dq(t) = 1$ or not.
Assume that the prices are set M equally-spaced times each day and that in the sample there are T days. Then, the intraday price difference or returns for day t are calculated as,

$$r_{t,j} = p_{t,j} - p_{t,j-1} \ where \ j = 1, \dots, M \ and \ t = 1, \dots, T \tag{4}$$

Note that working with a continuous electricity prices time series means that there are observations 24 hours per day 7 days per week without stopping. Hence, the first return for each day is the first quarter price of that day minus the last quarter price of the previous day [1]. Therefore, we are just losing the first observation of the whole sample.
When we work with high enough frequency data, it can be assumed that the drift of Equation (1) is equal to zero. Therefore, the RV for day t is the next one,

$$RV_t = \sum_{j=1}^{M} r_{t,j}^2. \tag{5}$$

It can be shown, see BNS (2004), that as the number of times that the prices are set each day goes to infinite, i.e. $M \to \infty$, RV converges to total quadratic variation,

$$\lim_{M\to\infty} \sum_{j=1}^{M} r_{t,j}^2 = \int_{t-1}^{t} \sigma^2(s)ds + \sum_{s=0}^{q(t)} \kappa^2(s).$$

In financial time series first-lag bipower variation is used as it is shown in BNS (2004, 2006). On the other hand, ABD (2007) and Ullrich(2012) suggest that for electricity time series first lag is not enough, because considering only one lag in bipower variation can make the jump detection to be downward biased. In this paper, we assume lags that go from 0 to 7. The bipower variation for day t for each lag is calculated as follows,

$$BV_t = \mu_1^{-2} \frac{M}{M-(1+i)} \sum_{j=1+(i+1)}^{M} |r_{t,j}||r_{t,j-(i+1)}| \ where \ i = 0, \dots, 7, \tag{6}$$

[1]However, working with financial time series as the market closes at a certain hour the calculations are made for each day t without including observations from the day before.

where $\mu_1 = E(|Z|) \equiv \sqrt{\frac{2}{\pi}}$ and Z denotes a white noise. In this case, BNS (2004) show that when $M \to \infty$ the bipower variation tends to the integrated variation,

$$\lim_{M\to\infty} BV_t = \int_{t-1}^{t} \sigma^2(s)ds,$$

As a consequence we have that $\lim_{M\to\infty} RV_t - BV_t = 0$ when there are no jumps. Using this fact Huang and Tauchen (2005) suggest the following statistic to detect significant jumps,

$$Z_t = \sqrt{M}\frac{(RV_t - BV_t)/RV_t}{\sqrt{(\mu_1^{-4} + 2\mu_1^{-2} - 5)\max[1, TQ_t/BV_t^2]}}. \tag{7}$$

The denominator of Equation (7) represents the effect of integrated quarticity which can be estimated using tripower quarticity (TQ). The tripower quarticity and its properties were suggested by Huang and Tauchen (2005). TQ_t for each day t and different lags is the next one,

$$TQ_t = \mu_{4/3}^{-3}\left(\frac{M^2}{M-2(i+1)}\right)\sum_{j=1+2(1+i)}^{M}(|r_{t,j}||r_{t,j-(i+1)}||r_{t,j-2(i+1)}|)^{4/3} \tag{8}$$

where,$i = 0, \ldots, 7$ and $\mu_{4/3} \equiv 2^{2/3}\Gamma(7/6)\Gamma(1/2)^{-1} \approx 0.8308609$.

The idea behind the Z_t statistic for jump detection is as follows. RV_t measures the RV and is the sum of the square of returns. Then, a rarely large value in the returns, a jump, will increase the value of the RV_t. On the other hand, BV_t measures the bipower variation and is calculated as the multiplication of absolute lagged returns. Jumps are not a common event, then an unusually large value in the returns is not followed by so large values, i.e. usually large values in returns are followed by normal values or negative jumps. Therefore, when we do the multiplication the value of BV_t does not increase so much. As a result, when there are large values in returns $RV_t - BV_t$ increases, then Z_t increases and day t is classified as jump day.

The structure of high frequency electricity prices has a peculiarity. Usually electricity prices show negative serial correlation in returns. Andersen et al. (2007) show that this negative autocorrelation causes the jump detection statistic in Equation (7) to be biased downward. This happens because a peak electricity price is usually followed by an off-peak jump or normal price, as a consequence the prices are serially negatively correlated. They suggest to increase the number of lags in returns, i.e. to increase i in the calculation of the BV and TQ, in order to avoid the jump detection to be downward biased.

Using the asymptotic distribution theory of BNS (2004) it can be proved that Z_t converges to a natural normal distribution when $M \to \infty$. Hence, if the Z_t statistic exceeds the critical value $\Phi_{1-\alpha}$ of natural normal distribution, day t is classified as jump day. For a chosen significance level α, jump component of volatility at day t is calculated as,

$$JV_t = I_{Z_t > \Phi_{1-\alpha}}(RV_t - BV_t), \tag{9}$$

where $I_{Z_t > \Phi_{1-\alpha}}$ is equal to 1 if $Z_t > \Phi_{1-\alpha}$, and 0 otherwise.
Once calculated the total RV and the jump component of the variation, JV, what remains is to calculate the continuous component of the total variation, CV,

$$CV_t = RV_t - JV_t. \tag{10}$$

Equation (10) is the continuous path of volatility, i.e. it measures the variation without taking into account jumps. Therefore, it is assumed to be less volatile.

3 Data

EPEX is the European Power Exchange platform for intra-day and day-ahead trading of electricity for the French, German, Austrian and Swiss energy markets. These four markets account for approximately a third of European electricity consumption. Although the volumes traded are not as big as in the day-ahead market, intraday market is very useful to adjust volumes to real-time demand. Within the intraday market, we focus on the intraday 15-minutes contracts continuous market for Germany and Austria, which is used to balance demand and supply. These prices are the ones we analyse because we are interested in high frequency data, so that the properties of the models hold.
In the intraday continuous market in Germany and Austria electricity is traded for a delivery the following 15-minutes. Each quarter hour, 15-minute periods can be traded until 45 minutes before delivery begins. Starting at 4pm on the current day, all 15-minute periods of the following day can be traded. The prices range from -9999 to 9999 and the minimum increment is 0.01 €/Mwh.

Figure 1: Volume of electricity traded in Germany and Austria (Twh).

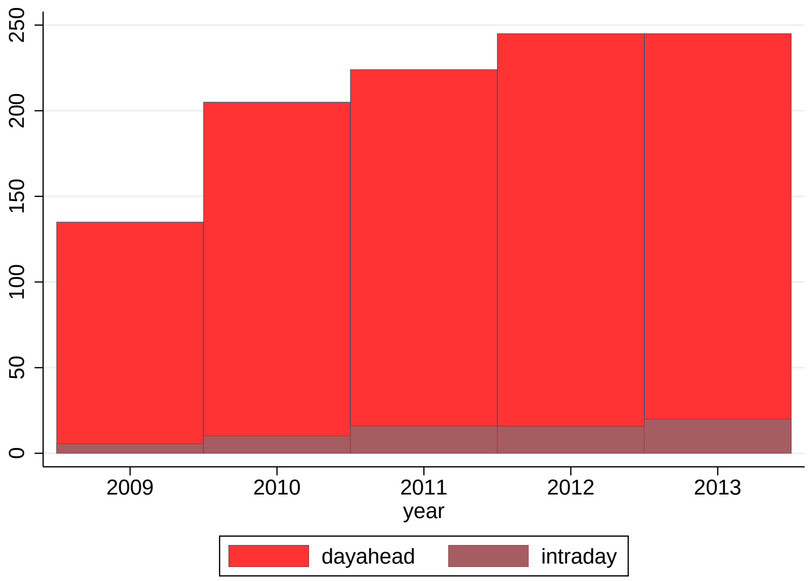


Figure 1 plots the volume of electricity traded in the intraday market and day-ahead market in Germany and Austria. As we can see the share of the intraday market is not very high and inside the intraday market the intraday continuous market of 15-minutes blocks is even smaller. Anyway, we are interested in this market because of the high frequency of the data, and 15-minutes is the highest frequency in the EPEX market.

Table 1: Share of electricity generation in Austria and Germany (%) by type in 2013.

Technology	Austria	Germany
Nuclear	0.00	19.25
Fossil	20.00	61.49
Renewable	10.00	16.04
Hydro	66.00	3.21
Others	4.00	0.00

These shares are taken from the annual report of the EPEX market 2013.

Figure 2: 15-minutes prices in the whole data.

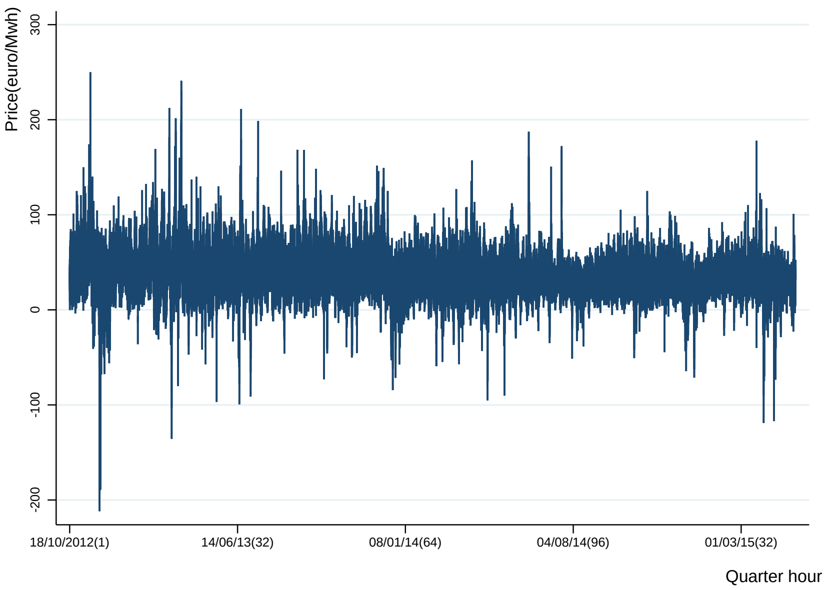


Table 1 reports the share of each type of electricity generation in the total electricity generation in Germany and Austria. As we can see the source of energy is quite different in both countries. The German electricity production relies heavily on fossil fuels, mainly lignite and hard coal, while the share of hydro is relatively small. However, Germany exhibits larger renewable generation (i.e. wind, solar, biomass and others) than Austria, probably due to the large scale deployment of the German on- and off-shore wind power industry. On the other hand, Austria produces most of the energy from hydro power plants which is very interesting as is relatively cheaper than the other electricity sources. It can be due to its proximity to big mountains where water flows the whole year. Austria has also a quite developed renewable energy production sector.

Our data starts in 18 November 2012 and goes to 7 May 2015. The 18 November 2012 is the first day because before this day intraday continuous market for 15-minutes contracts was not developed enough and these days were full of missing observations in Germany and Austria. Consequently, this fact makes more complicated the interpretation of the models explained in Section 2. The last day of the sample is the day we finished collecting the data.
As we can observed in Figure 2 the 15-minutes blocks present unusually big fluctuations that could be identified as jumps. So it suggests that the model that separates JV and CV can work better than the model using only the recently past observations of RV. It is also illustrated the high variation of the sample.
Figure 3 illustrates different patterns in the prices distinguishing between summer and

Figure 3: Mean prices in winter vs summer by quarter hour.

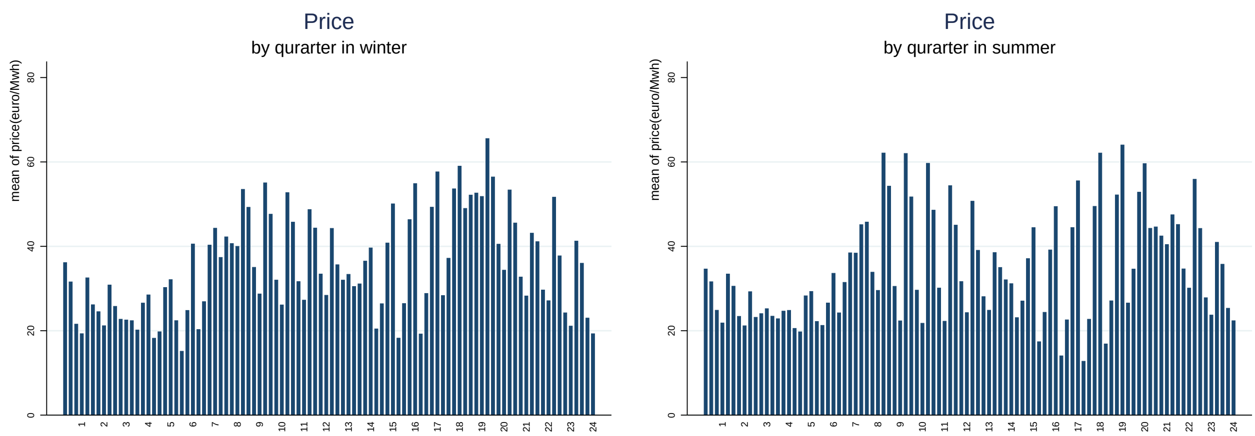
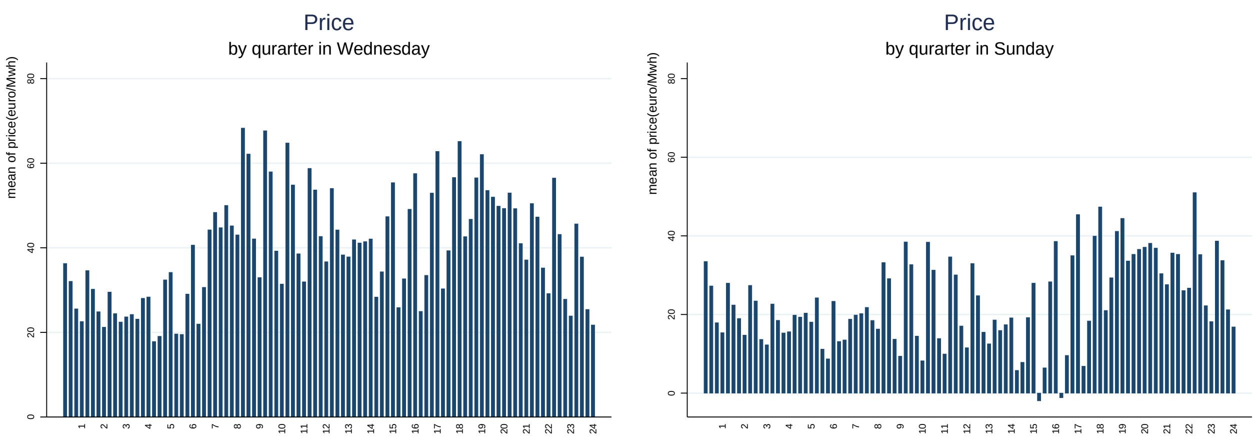


Figure 4: Mean prices in Wednesday vs Sunday by quarter hour.



winter, looking by quarter hour. In summer the demand is lower because many firms are closed. Therefore, the prices are lower comparing to winter when most of the firms are opened. In Figure 4 seasonal patterns are observed for the days of the week. On Sundays the demand is lower because in most of the factories Sundays are the free days, therefore, prices are lower than on Wednesdays. These patterns mentioned will be important to improve the model presented in Section 2.

Table 2: Descriptive statistics for prices in different seasons and days of the week.

	Wtr.	Spr.	Sum.	Aut.	Mn.	Tu.	Wd.	Th.	Fr.	St.	Sn.	Total
Minimum	-135.69	-117.06	-73.06	-211.84	-170.34	-211.84	-188.91	-67.98	-71.5	-117.06	-135.69	-211.84
Maximum	212.44	240.99	198.69	250.00	172.22	211.32	174.18	250.00	240.99	136.41	140.16	250.00
Mean	35.49	32.49	34.62	37.08	35.92	39.41	39.90	37.90	37.68	30.68	23.37	34.98
Median	35.33	31.23	32.73	36.40	34.78	33.95	37.89	35.88	35.96	30.69	24.57	33.95
St. Dev.	20.97	21.32	19.36	24.08	21.44	22.68	22.76	20.82	20.37	17.70	20.20	21.64
Skewness	0.17	0.70	0.85	-0.62	0.18	-0.24	-0.45	0.86	1.27	0.10	-0.75	0.14
Kurtosis	5.90	9.43	5.72	11.08	5.21	11.61	12.73	7.02	11.43	4.35	6.29	8.64

Wtr., Spr., Sum. and Aut stand for winter, spring, summer and autumn, respectively. On the other hand, Mo., Tu., Wd., Th., Fr., St and Sn are the days of the week starting from Monday. Finally, St. Dev. means standard deviation.

Table 2 reports descriptive statistics depending on the day of the week or the season of the year. These differences are not as big as expected but it is because the intraday 15-minutes contracts continuous market in Germany and Austria is only the $1-2\%$ of the whole market in these countries. Therefore, the prices do not change too much depending on those patterns. It is observed that comparing to the prices considering the whole sample in winter and autumn the prices are higher while in spring and summer are lower. In Germany and Austria winters are cold and all factories are opened, hence, the electricity demand is much higher comparing to summer and spring, in this last season there is a lot of water because it rains a lot and there is ice melting process in the mountains. Consequently, the hydro power generation, mainly in Austria, generates a lot of cheap electricity pulling down the prices. Regarding the variation it is observed that in spring and autumn prices are more volatile due to the changes in the weather.

Focusing on the days of the week, it is shown that in the working days of the week, from Monday to Friday, prices are higher than during the weekend. This happens because the factories are opened from Monday to Friday but then on Saturday some of them close and on Sundays many of the factories are closed, decreasing the demand and the prices. It is also observed that the volatility decreases during the weekend comparing to rest of the days of the week, because of the same arguments.

It is observed that the mean price is 34.98 €/MWh which is low, this can be because it is only a small market where the quantities traded are low. It is also shown that all prices present non zero skewness and leptokurtosis.

As mentioned previously, electricity prices changes are not mean zero, as a result the QVT based model must be slightly modified. The electricity prices present patterns that vary depending on the season of the year or day of the week as shown in Figure 2, thus we want the model to take into account these seasonality following Ullrich (2012). As returns present positive skewness, see Table 3, the median of the returns can be subtracted from the returns for each month of the year, each day of the week and each quarter hour of the day.

$$r^*_{t,j} = r_{t,j} - \hat{r}_{m,d,q},$$

where $\hat{r}_{m,d,q}$ is the median of the month (m), day of the week (d) and quarter hour (q).

Table 3: Descriptive statistics of returns and adjusted returns.

	r	r^*
Minimum	-178.94	-174.84
Maximum	194.00	211.6
Mean	-3.8 10^{-4}	-0.04
Median	-0.49	0.00
St. Dev.	18.94	12.11
Skewness	0.02	0.15
Kurtosis	7.21	16.38

r and r^* refer to returns and adjusted returns, respectively. St. Dev. means standard deviation.

Table 3 summarizes the descriptive statistics of returns and adjusted returns series. It is interesting to check how the variance diminishes when taking into account electricity prices patterns in the calculation of returns. It is important to underline that all of the time series present excess of kurtosis and positive skewness.

Figure 5: Correlogram of returns and adjusted returns.

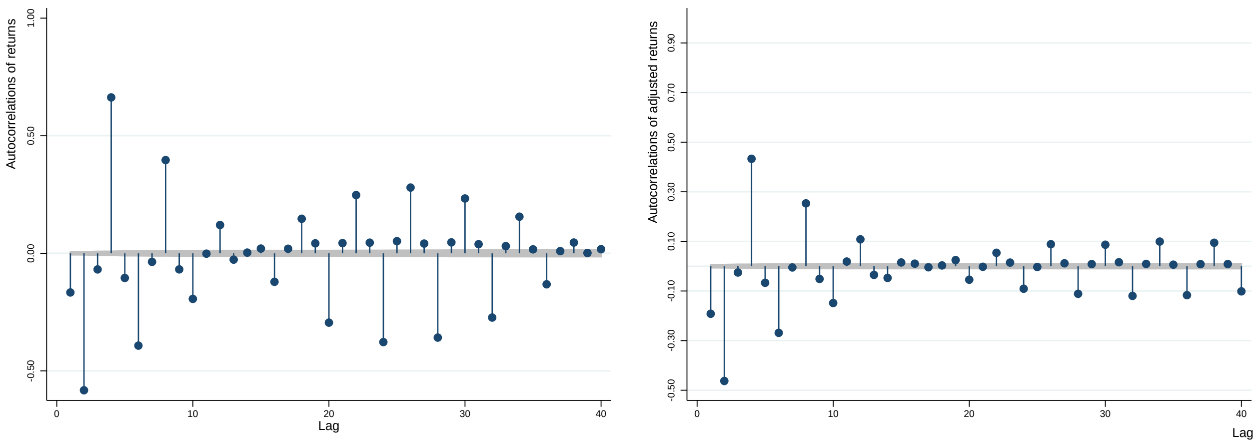


Figure 5 shows that in the first lags prices are negatively correlated as mentioned in Section 2. There exists high autocorrelation in returns but it decreases when we use adjusted returns. Therefore, these adjusted returns must be replaced in Equations (5), (6) and (8) as suggested by Ullrich (2012). This way, electricity price patterns are taken into account in the returns.

4 Realized volatility characteristics

In Section 2 how to calculate RV_t, CV_t and JV_t is explained, the last one positive for the days classified as jump days, and 0 for the rest of the days.

Table 4: Descriptive statistics of RV.

	RV	$\sqrt{RV}$	$\log RV$
Minimum	1730.76	41.60	7.46
Maximum	304580.3	551.89	12.63
Mean	14071.78	109.02	9.24
Median	9999.67	100.00	9.21
St. Dev	17023.86	46.79	0.72
Skewness	7.69	2.57	0.65
Kurtosis	103.58	12.94	0.83
Jarque-Bera	411644.77	7279.96	88.69

St. Dev. means standard deviation.

Table 4 presents descriptive statistics in the RV and two non linear transformations: logarithmic and square root. As expected, the standard deviation diminishes as more concave non linear forms are chosen. The concavity of the non linear forms makes high values of the time series diminish more than small values. Consequently, the jumps are not as notable as before using these non linear forms, becoming the time series smoother. The concavity of the logarithmic function is bigger than concavity of the square root function. Hence, the smoothest time series is the one with the logarithmic form. The three time series in Table 4 show positive skewness and leptokurtosis, these could imply that GARCH models can fit in these time series. It is also exposed that none of them is normally distributed, as confirmed by the Jarque-Bera statistic. An important fact is that all the time series have not 0 mean justifying the modification in QVT to calculate returns.

Following Section 2 we decompose RV into CV and JV. Table 5 reports descriptive statistics of each component and transformation. After careful scrutiny of Figure 5 the best jump detection is done choosing $i = 2$ and $\alpha = 0.01$. This way, we avoid the Z_t statistic to be downward biased, and we make the jump detection to be accurate enough choosing a pretty low significance level.

Table 5: Descriptive statistic CV and JV .

	CV	$\sqrt{CV}$	$\log CV$	JV	$\sqrt{JV}$	$\log JV$
Minimum	1267.10	35.90	7.14	463.66	21.53	6.14
Maximum	304580.30	551.89	12.63	56019.16	24.24	10.93
Mean	11416.91	97.71	9.02	4608.93	61.56	8.07
Median	7686.91	87.68	8.95	3094.01	55.62	8.04
St. Dev	15114.07	43.27	0.73	5751.96	28.65	0.80
Skewness	9.82	2.86	0.63	5.06	2.32	0.42
Kurtosis	163.33	17.17	0.94	34.07	8.72	0.67
Jarque-Bera	1016022.23	12295.09	93.48	5123.23	1135.51	52.23

CV is the continuous component and JV is the jump component of the variation. St. Dev. means standard deviation.

A similar pattern is observed when we take CV_t and JV_t. The more concave the function we choose the lower the standard deviation is. The standard deviation of the continuous component of the volatility is lower because we eliminate the jump component. These time series also show excess of kurtosis and positive skewness, as expected none of them is normally distributed, which indicates that the GARCH models can also fit when we separate the RV into jump component and continuous component of the variation. It is important to underline the high frequency of jumps in intraday continuous German and Austrian 15-minutes blocks market. In previous papers, like Chang et al (2008) or Ullrich (2012) the jump frequency was quite lower. In Chang et al (2008) it could be due to the downward bias in the estimation of the Z_t statistic, but in Ullrich (2012) different lags were assumed. The high frequency of jumps could be due to the small share of the whole electricity market in Austria and Germany.

Figure 6: Realized volatility and jump component of the variation.

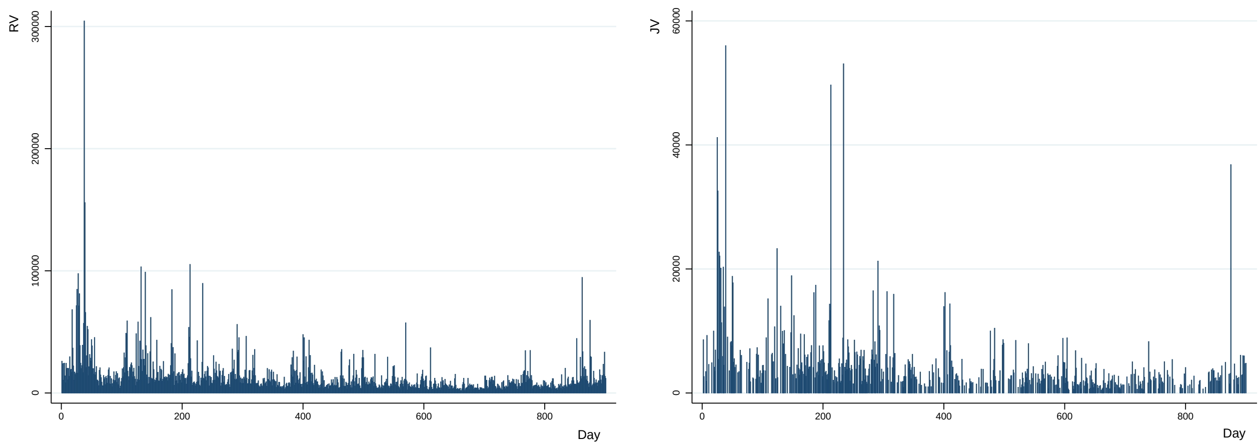


Figure 6 illustrates the RV and JV in the whole data. The jump frequency is 57.7%, choosing $i = 2$ and $\alpha = 0.01$, which means that out of 901 days 519 days are classified as jump days. It is observed that JV is discontinuous while RV is continuous. The variation is greater at the beginning of the sample than at the end of it. This occurs because in the beginning of the sample the intraday continuous market for 15-minutes contract was not as used as at the end, where the plants participating in this market are more habituated to trade in this market, i.e are usual participants in the 15-minutes blocks intraday continuous market in Germany and Austria.

5 Realized volatility forecasting models

It has been shown in Section 2 that the RV has two components, the continuous variation and the discontinuous variation or jump component. We test whether the predictions are better when we separate them or not.
The two non linear forms have been included. In the logarithmic model, as we have illustrated in the descriptive statistics the distribution of the time series is close to a log normal distribution, so, the normal distribution theory can be applied as ABDL (2003)

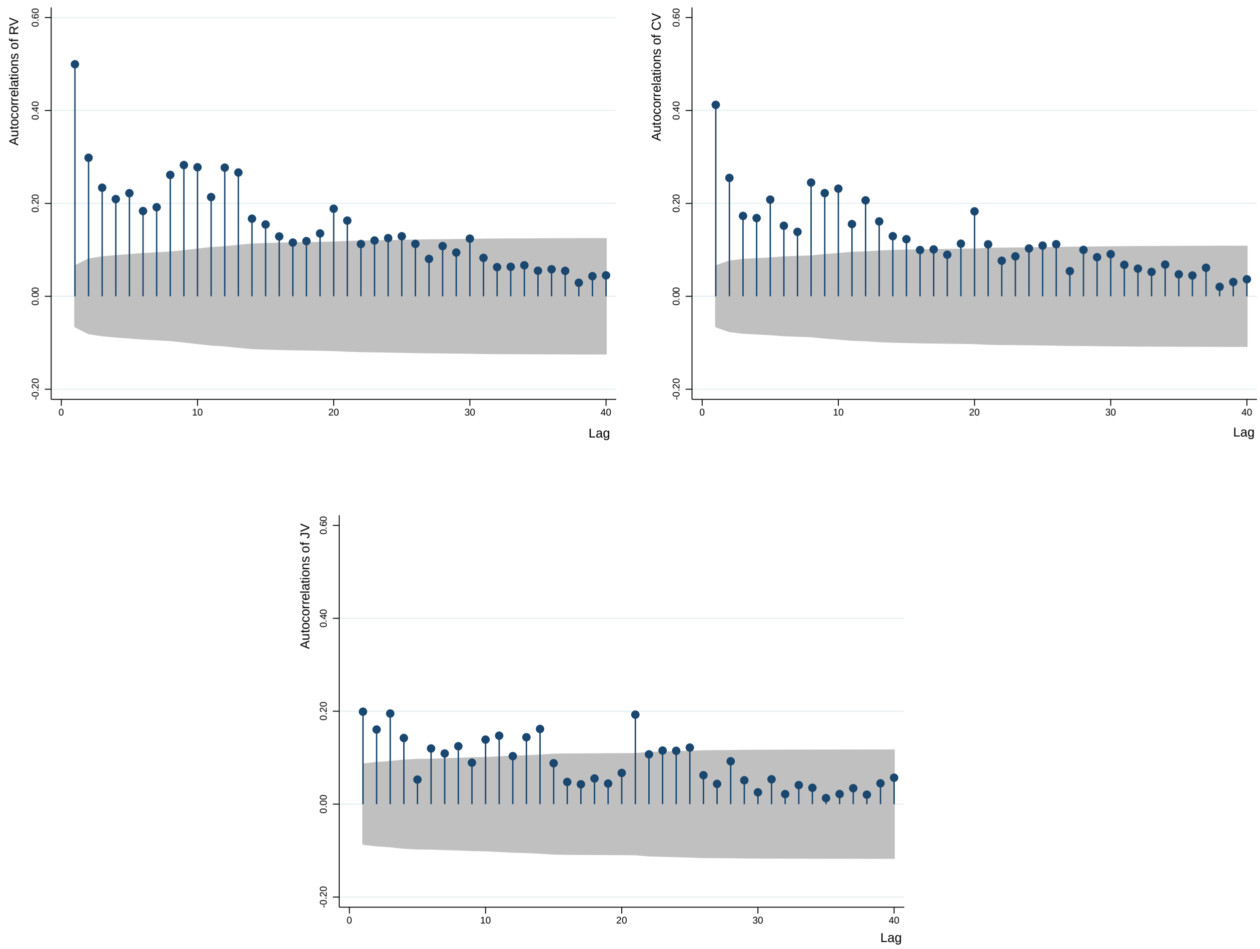
confirms. The second transformation the standard deviation form is the most common form used in previous research.
We start from the simplest model, the one introduced by Corsi (2004), so called HAR-RV model,

$$\sqrt{RV_t} = \beta_0 + \beta_1\sqrt{RV_{t-1}} + \beta_2\sqrt{RV_{w,t-1}} + \beta_3\sqrt{RV_{m,t-1}} + a_t, \tag{11}$$

$$\log RV_t = \beta_0 + \beta_1 \log RV_{t-1} + \beta_2 \log RV_{w,t-1} + \beta_3 \log RV_{m,t-1} + a_t, \tag{12}$$

where $RV_{w,t} = \frac{1}{7}\sum_{i=1}^{7} RV_{t-i}$ is the average RV in the previous seven days and $RV_{m,t} = \frac{1}{21}\sum_{i=1}^{21} RV_{t-i}$ is the average RV over the previous 21 days. In this model importance is given to the previous observations. Then following Corsi (2004), in order to have a parsimonious model the averages over the previous week and over the previous three weeks are taken. Looking at the correlogram of RV, Figure 7, it is clear that the autocorrelation is quite high until 21 days before, for that reason are chosen these averages. Therefore, the error term should not be autocorrelated.

Figure 7: Correlogram of RV, CV and JV.



The next step is to separate the RV into jump component and continuous component, i.e. from a HAR-RV model to a HAR-CV-JV model, as Chang et al. (2008) suggests,

$$\sqrt{RV_t} = \beta_0 + \lambda_1\sqrt{CV_{t-1}} + \lambda_2\sqrt{CV_{w,t-1}} + \lambda_3\sqrt{CV_{m,t-1}} + \\ + \theta_1\sqrt{JV_{t-1}} + \theta_2\sqrt{JV_{w,t-1}} + \theta_3\sqrt{JV_{m,t-1}} + a_t, \quad (13)$$

$$\log RV_t = \beta_0 + \lambda_1 \log CV_{t-1} + \lambda_2 \log CV_{w,t-1} + \lambda_3 \log CV_{m,t-1} + \\ + \theta_1 \log JV_{t-1} + \theta_2 \log JV_{w,t-1} + \theta_3 \log JV_{m,t-1} + a_t, \quad (14)$$

where $CV_{w,t} = \frac{1}{7}\sum_{i=1}^{7} CV_{t-i}$ and $JV_{w,t} = \frac{1}{7}\sum_{i=1}^{7} JV_{t-i}$ are the average price of the previous week of the continuous path and jump component of the variation, respectively. Similarly, $CV_{m,t-1} = \frac{1}{21}\sum_{i=1}^{21} CV_{t-i}$ and $JV_{m,t-1} = \frac{1}{21}\sum_{i=1}^{21} JV_{t-i}$ are the average price over the previous 21 days of CV and JV, respectively [2]. Figure 7 illustrates how after the lag 21 the autocorrelation diminishes a lot, that is the reason to choose 21 lags to take into account recent past[3].

Table 6 shows in first panel the estimation results of HAR-RV model. Looking at both non linear forms there is evidence of strong degree of volatility persistence in HAR-RV model, because all recent past observations are significant at 5% level of significance, with exception of the the week before average volatility in square root form which is significant at 10%. Using both transformations we confirm that a positive RV is estimated to be followed by a positive volatility the day after, the week after and three weeks after, as all estimated coefficients are positive. A curiosity occurs in these estimations in both non linear forms, $0.23 > 0.14$ in the square root form and $0.22 > 0.16$ in the logarithmic form, i.e. estimated effect of the average three week volatility on RV_t is larger than that oh the week component.

In the second panel of Table 6 the estimation results of the HAR-CV-JV model are reported. The previous observations of the CV and JV are highly significant in both non linear forms. Using the logarithmic transformation, as the concavity is higher than using the standard deviation form, the recent past observations of CV are more important and significant at 5% while looking at the jump component only the previous observation is significant. Looking at the standard deviation form only the average of the previous three weeks of CV is significant at 10% and only the previous observation is significant for JV. As similar phenomenon than in the HAR-RV model occurs in the HAR-CV-JV models. Focusing on CV, the effect on the current RV of the previous 21 days average is greater than the effect of the previous week average using both transformations, as $0.18 > 0.09$ in the square root form and $0.17 > 0.13$ in the logarithmic form.

In the HAR-RV model looking at the previous observation the estimated coefficient is 0.44 taking the square root transformation whereas in the HAR-CV-JV model the estimated

[2] As suggested in Section 4 in the jump detection $i = 2$ and $\alpha = 0.01$ are used.

[3] Regarding no jumps days when the logarithmic transformation is done as $\log 0$ does not exist, we have substituted $\log 0$ by 0, in order not to loose observations.

Table 6: HAR-RV and HAR-CV-JV estimation results.

		$\sqrt{RV}$	$\log RV$
HAR-RV			
RV_{t-1}	β_1	0.44***	0.45***
$RV_{w,t-1}$	β_2	0.14*	0.16**
$RV_{m,t-1}$	β_3	0.23***	0.22***
Intercept	β_0	18.86***	1.45***
adj-R^2		38.06 %	43.05 %
HAR-CV-JV			
CV_{t-1}	λ_1	0.42***	0.44***
$CV_{w,t-1}$	λ_2	0.09	0.13**
$CV_{m,t-1}$	λ_3	0.18*	0.17**
JV_{t-1}	θ_1	0.21***	0.03***
$JV_{w,t-1}$	θ_2	0.18	0.03
$JV_{m,t-1}$	θ_3	0.09	0.05
Intercept	β_0	19.33***	1.78***
adj-R^2		38.70 %	43.41 %

The estimated coefficients of equations HAR-RV, HAR-CV-JV models for both non linear forms. $***$, $**$ and $*$ illustrate significance at the 1%, 5% and 10% levels, respectively. And adj-R^2 is the adjusted R^2.

coefficient of the previous observation is $\hat{\lambda}_1 + \hat{\theta}_1 = 0.63$, hence in the HAR-CV-JV model the previous observation has greater effect on the RV_t than in the HAR-RV model. On the other hand, in the logarithmic form the estimated coefficients of the previous observation are similar in both models, 0.45 in the HAR-RV model and $\hat{\lambda}_1 + \hat{\theta}_1 = 0.47$ in the HAR-CV-JV model.

5.1 Generalized Autoregressive Conditional heteroskedasticity (GARCH) structures

In this subsection we explain briefly GARCH structures. For further information any time-series book can be checked, for example, Tsay (2005).
The idea is to analyse electricity prices time series, high frequency time series, where usually there is time variant conditional second order moment, consequently the ARIMA processes cannot use all the information available about the time series. Time series with this characteristic are better explained using GARCH structures which try to capture the observed trend in the volatility. Among the large variety of GARCH structures we use GARCH(1,1) and EGARCH(1,1), because these are some the best explaining the electricity price volatility, for instance, Chang et al (2008) use EGARCH (1,1) structures to explain Austrian electricity prices volatility.
The first step is to decide whether GARCH structures must be introduced in the models or not. Two conditions must fulfil a_t (error term in equations 11 to 14) to justify the

use of GARCH structures: first, a_t must not be autocorrelated, but has to be dependent. Therefore, the time series a_t^2 has to be autocorrelated which is the second condition.
The Ljung-Box $Q(q)$ statistic is used to test whether the time series present autocorrelation of order q or not. The Ljung-Box statistic is calculated as follows,

$$Q(q) = T(T+2)\sum_{j=1}^{q} \frac{\rho_j}{T-j}, \tag{15}$$

where T is the number of observations in the sample, and ρ_j is the autocorrelation coefficient of order j. On the other hand, q is the order and must be defined, usually is chosen looking at the correlogram of the time series.
Applying the Ljung-Box test the null hypothesis states that the first q lags of the time series a_t present no autocorrelation. Under this null hypothesis the statistic is asymptotically distributed as a χ^2_{q-k} where k is the number of parameters estimated.
Finally, to justify the use of a GARCH structure in the time series a_t the null hypothesis must not be rejected, i.e. there is no autocorrelation in a_t, and in the time series a_t^2 the null hypothesis must be rejected, i.e. the time series a_t^2 is autocorrelated.
If we want to be sure about the use of GARCH structures it is interesting to calculate the $Q(q)$ for different orders of q.
If it is necessary to include GARCH structure in the innovations of HAR-RV and HAR-CV-JV models the next step is to choose the one that fits better the time series. In this paper, among the vast variety of GARCH structures the GARCH(1,1) and the asymmetric EGARCH(1,1) are considered which are explained below.

GARCH(1,1)

The GARCH(1,1) model is given by,

$$\begin{aligned} a_t &= \sigma_t \epsilon_t, \\ \sigma_t^2 &= \alpha_0 + \alpha_1 a_{t-1}^2 + \delta_1 \sigma_{t-1}^2, \end{aligned} \tag{16}$$

where ϵ_t is assumed to be normally distributed with mean 0 and standard deviation 1 or it can be assumed another distribution such as the t-student, which has heavier tails than normal distribution. Conditions $\alpha_0 > 0$, $\alpha_1 \geq 0$, $\delta_1 \geq 0$ and $\alpha_1 + \delta_1 < 1$ ensure that the variance is positive and the process is stationary.

EGARCH(1,1)

The GARCH(1,1) does not take into account the asymmetric effects observed in the time series. On the contrary, the asymmetric EGARCH (Exponential GARCH) model fulfils this property. The EGARCH(1,1) model is the next one,

$$a_t = \sigma_t \epsilon_t,$$

$$\log \sigma_t^2 = \alpha_0 + \alpha_1 \frac{|a_{t-1}|}{\sigma_{t-1}} + \delta_1 \log \sigma_{t-1}^2 + d_1 \frac{a_{t-1}}{\sigma_{t-1}}, \tag{17}$$

where as before ϵ_t follows standard normal distribution or $t - student$ distribution. In this model it is not required the coefficients to be positive, even if $\log \sigma_t^2 < 0$ when exponentials are taken $\sigma_t^2 = e^{\log \sigma_t^2} > 0$.This is only one way to write down EGARCH(1,1) structures there are many other ways which are analogous.
In this new model the reaction of a positive or negative shock is stored in the coefficient d_1. Hence, if $d_1 < 0$, there is leverage effect, this means that a negative shocks have bigger effect in the time series than positive shocks. If $d_1 > 0$, there is inverse leverage effect, which is just the opposite positive shocks have bigger effect in the time series than negative shocks.
When GARCH structures are included in the model maximum likelihood estimation is used, and GARCH structures are correctly introduced in the model when the standardized errors follow a white noise process.

5.2 HAR-GARCH-RV, HAR-GARCH-CV-JV, HAR-EGARCH-RV and HAR-EGARCH-CV-JV models

Table 7: The Ljung-Box Q(q) statistic for different lags.

		5 lags		10 lags		15 lags		20 lags	
		a_t	a_t^2	a_t	a_t^2	a_t	a_t^2	a_t	a_t^2
$\log RV$	HAR-RV	3.28	9.23	5.77	14.36	13.85	17.82	25.51	20.69
	HAR-CV-JV	3.69	9.12	6.14	13.12	14.79	17.59	26.82	21.06
$\sqrt{RV}$	HAR-RV	2.32	13.26***	10.44	39.31***	23.18*	53.867***	31.01*	55.27***
	HAR-CV-JV	2.47	13.35**	10.31	35.29***	24.70	53.18***	32.09**	54.65***

The Ljung-Box autocorrelation test is shown for the time series a_t in Equations (12), (14), (11) and (13), respectively. $***$, $**$ and $*$ illustrate significance at the at 1%, 5% and 10% levels, respectively.

After looking at the HAR-RV and HAR-CV-JV models the next step is to check whether these models justify the use of GARCH structures or not in each of the non linear transformations of the RV.
Table 7 reports the Ljung-Box statistic for a_t and a_t^2 considering different number of lags, i.e. different q. Focusing on the logarithmic transformation, it is clear that the error term of both HAR-RV and HAR-CV-JV models is serially uncorrelated at 5% level of significance and that a_t^2 is serially uncorrelated at 5% level of significance, as well. Therefore, GARCH structures are not justified in the logarithmic transformation. This is an expected result because, as mentioned in Section 2, taking the logarithm of a volatile time series a log-normal distribution is what usually remains.
Looking at the standard deviation form there is strong evidence to justify GARCH structures in both HAR-RV and HAR-CV-JV models. On the one hand, the error terms are serially uncorrelated at 5% level of significance, with exception of between 20 lags and 30 in the HAR-CV-JV models. Although, the null hypothesis that the serial autocorrelation is rejected at 5% level of significance taking between 20 and 30 lags, the lowest $p - value$ is equal to 0.042 then the null hypothesis is not rejected at 4% significance level. So, we

Table 8: HAR-GARCH-RV, HAR-EGARCH-RV, HAR-GARCH-CV-JV and HAR-EGARCH-CV-JV estimation results.

HAR-GARCH-RV			HAR-EGARCH-RV		
RV_{t-1}	β_1	0.45^{***}	RV_{t-1}	β_1	0.48^{***}
$RV_{w,t-1}$	β_2	0.17^{**}	$RV_{w,t-1}$	β_2	0.10^{***}
$RV_{m,t-1}$	β_3	0.21^{***}	$RV_{m,t-1}$	β_3	0.26^{***}
Intercept	β_0	16.35^{***}	Intercept	β_0	14.98^{***}
a_{t-1}^2	α_1	0.11^{***}	$\|a_{t-1}\|/\sigma_{t-1}$	α_1	-0.05^{***}
σ_{t-1}^2	δ_1	0.84^{***}	$\log \sigma_{t-1}^2$	δ_1	0.96^{***}
			$\|a_{t-1}\|/\sigma_{t-1}$	d_1	0.18^{***}
Constant	α_0	65.70^{***}	Constant	α_0	0.28^{***}
Log likelihood		-4331.84	Log Likelihood		-4285.56

HAR-GARCH-CV-JV			HAR-EGARCH-CV-JV		
CV_{t-1}	λ_1	0.43^{***}	CV_{t-1}	λ_1	0.50^{***}
$CV_{w,t-1}$	λ_2	0.11	$CV_{w,t-1}$	λ_2	0.16^{***}
$CV_{m,t-1}$	λ_3	0.21^{***}	$CV_{m,t-1}$	λ_3	0.14^{***}
JV_{t-1}	θ_1	0.19^{***}	JV_{t-1}	θ_1	0.17^{***}
$JV_{w,t-1}$	θ_2	0.20^{*}	$JV_{w,t-1}$	θ_2	-0.09^{***}
$JV_{m,t-1}$	θ_3	0.002	$JV_{m,t-1}$	θ_3	0.25^{***}
Intercept	β_0	16.71^{***}	Intercept	β_0	15.42^{***}
a_{t-1}^2	α_1	0.11^{***}	$\|a_{t-1}\|/\sigma_{t-1}$	α_1	-0.07^{***}
σ_{t-1}^2	δ_1	0.85^{***}	$\log \sigma_{t-1}^2$	δ_1	0.96^{***}
			$\|a_{t-1}\|/\sigma_{t-1}$	d_1	0.19^{***}
Constant	α_0	62.94^{***}	Constant	α_0	0.30^{***}
Log likelihood		-4329.25	Log likelihood		-4280.20

The squared root form is analysed. HAR-GARCH-RV and HAR-EGARCH-RV, HAR-GARCH-CV-JV and HAR-EGARCH-CV-JV models are estimated. $***$, $**$ and $*$ illustrate significance at the at 1%, 5% and 10% levels, respectively.

assume that both error terms are serially uncorrelated. On the other hand, the square of the error terms are highly autocorrelated in all the different lags considered. Hence, GARCH structures are justified in the HAR-RV and HAR-CV-JV models in the standard deviation form.

GARCH(1,1) and EGARCH (1,1) structures are introduced in the HAR-RV and HAR-CV-JV models in the standard deviation form. The new models are called HAR-GARCH-RV, HAR-EGARCH-RV, HAR-GARCH-CV-JV and HAR-EGARCH-CV-JV[4].

As it is illustrated in Table 8 the estimated coefficients of GARCH(1,1) models, (Equation 16), fulfil the properties mentioned in Subsection 5.1, in both HAR-GARCH-RV

[4]Working with GARCH(1,1) structures there are two possible distributions, as explained above, in this case better results are obtained using the normal distribution, because Ljung-Box statistic values are lower in every number of lags considered. Therefore, in the GARCH(1,1) model normal distribution is used.

Among all possible EGARCH(1,1) structures best results are obtained using the the asymmetric normally distributed EGARCH(1,1) model, using the Ljung-Box statistic to choose the best model as before.

and HAR-GARCH-CV-JV models, in order to ensure the time-series is stationary. It also shows that the estimated coefficients are similar in the HAR-RV model and HAR-GARCH-RV models, and in the HAR-CV-JV and HAR-GARCH-RV models, as well.
There are also provided the estimations of the HAR-EGARCH-RV and HAR-EGARCH-CV-JV models. Focusing on the asymmetry coefficient (d_1), an inverse leverage effect is observed, which means that positive shocks have bigger effect in RV_t time series than negative shocks. The fact that working with electricity markets the inverse leverage arises has been suggested by authors like ABD (2007). As occurs with GARCH structures using EGARCH structures the remaining coefficients' estimation are similar to the estimations in the HAR-RV and HAR-CV-JV models.
Now the goodness of fit for the models is analysed. We check whether standardized errors and their squares follow a white noise process.

Table 9: The Ljung-Box Q(q) statistic for different lags.

	5 lags		10 lags		15 lags		20 lags	
	ϵ_t	ϵ_t^2	ϵ_t	ϵ_t^2	ϵ_t	ϵ_t^2	ϵ_t	ϵ_t^2
HAR-GARCH-RV	5.52	3.00	8.49	10.84	15.28	15.78	23.23	16.66
HAR-GARCH-CV-JV	5.95	2.63	8.85	10.06	15.81	16.01	23.95	16.97
HAR-EGARCH-RV	5.05	4.19	8.10	12.29	16.00	17.97	24.93	18.46
HAR-EGARCH-CV-JV	5.48	3.81	8.11	11.04	16.38	17.07	24.56	17.47

The Ljung-Box autocorrelation test of the standardized error term of the models HAR-GARCH-RV, HAR-GARCH-CV-JV, HAR-EGARCH-RV and HAR-EGARCH-CV-JV, respectively. $***$, $**$ and $*$ illustrate significance at 1%, 5% and 10% levels, respectively.

Table 9 shows that the standardized error terms and their squares follow a white noise process for the four models. Therefore, the GARCH(1,1) and EGARCH(1,1) fix correctly the data working with standardized deviation form.

6 Forecast

The aim of this paper is to find out the best model to predict electricity prices volatility in the Austrian and Germany in the 15-minutes blocks intraday continuous market, so the most relevant results will be presented in this section. We have introduced many different models in the previous section, the predictive power of each of the models must be measured. In-sample and out-of-sample criteria will be used for that end.
The best model in terms of prediction of the data is the one with the lowest value of any criteria. As these criteria work with error terms, to have the lowest value would mean that the error is the smallest one, hence, it is the best model for any criterion.

In-sample criteria

Firstly, in-sample criteria are used. This means that all the sample observations are considered in the estimation of the models and then the error term is calculated to decide which model is the best one. We use the AIC (Akaike information criterion) and BIC (Bayesian information criterion) criteria. These criteria are only used to compare maximum likelihood estimations, hence, we only compare within HAR-GARCH-RV, HAR-GARCH-CV-JV, HAR-EGARCH-RV and HAR-EGARCH-CV-JV models in the standard deviation form.

$$AIC = -2LogLikelihood + 2k \tag{18}$$

$$BIC = -2LogLikelihood + \log(T)k \tag{19}$$

where k is the number of parameters estimated and T is the number of observations [5].

Table 10: In-sample criteria to compare models estimated using maximum likelihood.

$\sqrt{RV}$	AIC	BIC
HAR-GARCH-RV	8677.67	8711.122
HAR-GARCH-CV-JV	8678.50	8726.28
HAR-EGARCH-RV	8587.13	8625.36 $\star$
HAR-EGARCH-CV-JV	8582.49$\star$	8634.97

AIC and BIC criterion are shown in the square root form.
Find marked with an $\star$ the best result prediction.

In Table 10 the AIC and BIC criteria are shown for the standard deviation for the HAR-GARCH-RV, HAR-EGARCH-RV, HAR-GARCH-CV-JV and HAR-EGARCH-CV-JV models. Illustrated in Table 10 is that in the square root transformation the results are not consistent, according to the AIC criterion the best model is the HAR-EGARCH-CV-JV model and according to the BIC criterion the best model id the HAR-EGARCH-RV. Therefore, as more consistent results are required other criteria are used.
In order to be able to compare OLS estimations we use the adjusted-R^2 to compare the different models. In the maximum likelihood estimation the log likelihood that remains can be another useful criterion to decide which model has the best prediction.
It is observed in Table 6 that there is a slight improvement, when we separate the RV into CV and JV, in the explanatory power of the model as the adjusted-R^2 is a bit higher. Although the explanatory power is inconspicuous, the logarithmic transformation explains better the variation of the HAR-RV model as the adjusted-R^2 is higher.
Looking at the Log Likelihood functions in Table 8 the maximum is greater when the partition in RV is done. Therefore, better result remain when RV is separated into continuous variation and jump component of the variation. Looking at the results the best model within the models that are estimated using maximum likelihood is the HAR-EGARCH-CV-JV, as the maximum likelihood is achieved. It is important to underline that in all cases is better to separate the RV into CV and JV.

Out-of-sample criteria

[5] In the AIC and BIC criteria the models that separate RV into CV and JV are more penalized because a larger number of parameters is estimated.

The first out of sample criteria used are the next ones: MAE (mean absolute error), RMSE (root mean square error) and MAPE (mean absolute percentage error) criteria. As the name says these are out-of-sample criteria, so we use the first 774 days of our sample, from 18th of November 2012 to 31st of December 2014, to estimate the models. Then, we use these estimates to predict the values of the RV for the rest of the 127 days, from 1st of January 2015 to 7th of May 2015, and we compare these values to the observed ones of the RV for these days.

$$MAE = \frac{1}{N}\sum_{t=1}^{N} |RV_t - \hat{RV_t}| \tag{20}$$

$$RMSE = \sqrt{\frac{1}{N}\sum_{t=1}^{N}(RV_t - \hat{RV_t})^2} \tag{21}$$

$$MAPE = \frac{1}{N}\sum_{t=1}^{N} \frac{|RV_t - \hat{RV_t}|}{RV_t} \tag{22}$$

where N is the number of forecasted observations, in our case 127. $\hat{RV_t}$ is the predicted value of RV using the models described in Section 5.

Table 11: MAE, RMSE and MAPE criteria.

$\log RV$	MAE	RMSE	MAPE
HAR-RV	0.38	0.53	$5.1\ 10^{-4}$
HAR-CV-JV	0.38⋆	0.53⋆	$4.9\ 10^{-4}$⋆

$\sqrt{RV}$	MAE	RMSE	MAPE
HAR-RV	20.91	33.54	$2.4\ 10^{-3}$
HAR-CV-JV	20.17⋆	33.29⋆	$2.3\ 10^{-3}$⋆
HAR-GARCH-RV	20.64	33.55	$2.3\ 10^{-3}$
HAR-GARCH-CV-JV	20.76	33.52	$2.4\ 10^{-3}$
HAR-EGARCH-RV	20.94	33.84	$2.4\ 10^{-3}$
HAR-EGARCH-CV-JV	20.97	34.13	$2.3\ 10^{-3}$

MAE is the mean absolute error, RMSE means the root mean squared error and MAPE is the mean absolute percentage error. Only two decimals are shown if the first two decimals are equal the third one is compared but it is not reported. In the first panel the logarithmic transformation is analysed and in the second panel the standard deviation form. Find marked with an ⋆ the best selection.

In Table 11 first panel shows the MAE, RMSE and MAPE criteria for the logarithmic form. The best model to the logarithmic form is the HAR-CV-JV model. This is a consistent result, as the three criteria select the HAR-CV-JV as the best model to forecast

RV.
Looking at the second panel of Table 11 there are the MAE, RMSE and MAPE criteria for the standard deviation form. In the standard deviation forms, also, a pretty consistent result is obtained, the best model is the HAR-CV-JV.
Finally, the last criterion we use is the Andersen et al (2003) approach. This is an out-of-sample criterion pretty different form, as it is not based on the prediction error measures. This is a regression based criterion, i.e. using the predicted values of RV from 1st January 2015 to 7th of May 2015 for two different models a regression it is run where the dependent variable is the observed value of the RV for each of the days.

$$RV_t = \beta_0 + \beta_1 \widehat{RV}_{t,model1} + \beta_2 \widehat{RV}_{t,model2} + \epsilon_t \tag{23}$$

where RV_t is the observed value of the time series, $\widehat{RV_{t,model1}}$ is the prediction of model 1 while $y_{\widehat{RV,model2}}$ is the prediction of model 2. ϵ_t is a normal distributed error.
Therefore, using this approach the models are compared in pairs. Firstly, we analyse the logarithmic transformations. There are only two models in this case.

Table 12: The Andersen approach for the logarithmic transformation.

HAR-RV	β_1	0.06
HAR-CV-JV	β_2	0.93^{***}
Intercept	β_0	0.10

The Andersen approach is used only once $\log RV_t = \beta_0 +$ $+\beta_1 \log \widehat{RV}_{t,HAR-RV} +$ $+\beta_2 \log \widehat{RV}_{t,HAR-CV-JV} + \epsilon_t$. $***$ illustrates significance at 1%.

As shown in Table 12 the best model is the HAR-CV-JV model supporting the results obtained by MAE, RMSE and MAPE criteria.
Secondly, the standard deviation form is analysed. In this case, 6 different models are compared in pairs.

Table 13 shows clearly that in all cases is better to split the RV into CV and JV at 5 % level of significance, with exception of the HAR-EGARCH-RV and HAR-EGARCH-CV-JV where the former model is preferred. Illustrated in the first row the fact that the HAR-CV-JV model is preferred to the HAR-RV model. In the third one, it is observed that the HAR-CV-JV model predicts better the RV than the HAR-GARCH-CV-JV, where the last model is better than the HAR-GARCH-RV model. In a similar way, the last row shows that the HAR-CV-JV and HAR-GARCH-CV-JV are preferred to the HAR-EGARCH-CV-JV model, and as mentioned before the HAR-EGARCH-RV model is also better than the HAR-EGARCH-CV-JV model. Comparisons between the HAR-RV, HAR-GARCH-RV and HAR-EGARCH-RV models cannot be done.

Table 13: The Andersen approach for the square root transformation.

	HAR-RV	HAR-CV-JV	HAR-GARCH-RV	HAR-GARCH-CV-JV	HAR-EGARCH-RV
HAR-CV-JV	β_1 1.43*** β_2 -0.45 Intercept 2.18				
HAR-GARCH-RV	β_1 3.14 β_2 -2.21 Intercept 9.49				
HAR-GARCH-CV-JV		β_1 -0.19 β_2 1.18** Intercept 1.17	β_1 1.24*** β_2 -0.27 Intercept 2.41		
HAR-EGARCH-RV	β_1 -0.24 β_2 1.25 Intercept -1.01		β_1 -0.20 β_2 1.19 Intercept 2.55		
HAR-EGARCH-CV-JV		β_1 -0.03 β_2 1.02*** Intercept 1.02		β_1 0.09 β_2 0.89*** Intercept 3.02	β_1 0.20 β_2 0.72* Intercept 9.14

The Andersen approach is used, in each of the cases the β_1 makes reference to the model in the row and the β_2 makes reference to the model in the column, for instance, in the first row first column it is shown the $\sqrt{RV_t} = \beta_0 + \beta_1\sqrt{\widehat{RV}_{t,HAR-CV-JV}} + \beta_2\sqrt{\widehat{RV}_{t,HAR-RV}} + \epsilon_t$ regression. $***$, $**$ and $*$ illustrate significance at 1%, 5% and 10% levels, respectively.

Summing up, the Andersen approach confirms that the HAR-CV-JV model makes the best predictions in the standard deviation form, as well. Giving more consistence to the result of the MAE, RMSE and MAPE criteria.

7 Conclusions

In this paper, the RV of the electricity prices in the intraday continuous market for 15-minutes contracts in Germany and Austria is modelled. To that end, the quadratic variation theory is used, giving a non parametric jump detection.
Once established the RV and jump detection, considering different lags in the jump detection, two models are developed. One taking into account only the previous observations of the RV and the second one separating the RV into the jump component and continuous path, considering to non linear transformations the square root and the logarithm. Then, GARCH structures are included in both models considering the two non linear transformations.
Although the market we analyse is small comparing to the whole Austrian and German electricity market, we found quite interesting facts.
We get a very high frequency of jump, 57.7% of the days in the sample are classified as jump days. Therefore, it is not surprising that better forecasts are achieved making the partition of RV into CV and JV.
Another fact is that in this market GARCH structures do not improve the RV forecasts at all. Considering the logarithmic form GARCH structures are not even justified whereas using the standard deviation form GARCH structures are justified but the predictions are not improved.
Therefore, in this market to predict the volatility are not needed complicated models

including GARCH structures. The best forecasts are using simpler HAR-CV-JV models for the square root and logarithmic transformations.
Further research must be done in the EPEX market. As the EPEX market is the main electricity market in four countries, it would be interesting to compare different countries like Germany and France, two countries where the source of energy is quite different. In Germany mainly fossil fuels, and in France mainly nuclear power. In these countries there is intraday continuous market in 1-hour blocks. Although the frequency is lower, the models developed in this paper can work well also on that frequency and the volatility of both countries could be compared.
Some improvements could also be included in the models. As the renewable energy is the most sustainable energy source, this generation sector is in development in many countries, for instance, Germany and Austria. This energy source depends on the weather, hence, weather forecasts could be included in the models in order to get better predictions. This is a very interesting suggestion but the problem is that in many countries this data is not available or it has to be paid.

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